# Coimisiún na Scrúduithe Stáit
State Examinations Commission

# Leaving Certificate 2020

# Marking Scheme

# Mathematics

# Higher Level

**Note to teachers and students on the use of published marking schemes**

Marking schemes published by the State Examinations Commission are not intended to be standalone documents.  They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme.  This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme.  The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme.  This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner.  The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents.  While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year.  This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive.  Variations and alternatives may also be acceptable.  Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

**Future Marking Schemes**

Assumptions about future marking schemes on the basis of past schemes should be avoided.  While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year.  The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year.  Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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## Paper 1

## Paper 2

# Marking Scheme – Paper 1, Section A and Section B
## Structure of the marking scheme
Candidate responses are marked according to different scales, depending on the types of response anticipated.  Scales labelled A divide candidate responses into two categories (correct and incorrect).  Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on.  The scales and the marks that they generate are summarised in this table:

| Scale label | A | B | C | D | E |
|---|---|---|---|---|---|
| No of categories | 2 | 3 | 4 | 5 | 6 |
| 5 mark scales | 0, 5 | 0, 2, 5 | 0, 3, 4, 5 | | |
| 10 mark scales | 0, 10 | 0, 5, 10 | 0, 4, 8, 10 | 0, 3, 5, 8, 10 | |
| 15 mark scales | 0, 15 | 0, 7, 15 | 0, 5, 10, 15 | 0, 4, 7, 11, 15 | |
| 20 mark scales | 0, 20 | 0, 10, 20 | 0, 7, 13, 20 | 0, 5, 10, 15, 20 | |
| 25 mark scales | 0, 25 | 0, 12, 25 | 0, 8, 17, 25 | 0, 6, 12, 19, 25 | 0, 5, 10, 15, 20, 25 |

A general descriptor of each point on each scale is given below.  More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.
## Marking scales – level descriptors
### A-scales (two categories)
- incorrect response
- correct response
### B-scales (three categories)
- response of no substantial merit
- partially correct response
- correct response
### C-scales (four categories)
- response of no substantial merit
- response with some merit
- almost correct response
- correct response
### D-scales (five categories)
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response
### E-scales (six categories)
- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

**NOTE:** In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded.
Rounding and units penalty to be applied only once in each section (a), (b), (c) etc.
Throughout the scheme indicate by use of * where an arithmetic error occurs.

## Summary of mark allocations and scales to be applied

### Section A

Question 1
(a)(i)      10C
(a)(ii)     5C
(a)(iii)    5C
(b)         5D

Question 2
(a)         5D
(b) (i)     5C
(b) (ii)    15D

Question 3
(a)         10C
(b)(i)      5C
(b)(ii)     10D

Question 4
(a)         15D
(b)         5D
(c)         5C

Question 5
(a)         15C
(b)         10D

Question 6
(a)         15C
(b)(i)      5C
(b)(ii)     5D

### Section B

Question 7 (50 Marks)
(a)(i)      10C
(a)(ii)     5C
(b)(i)      5C
(b)(ii)     5C
(b)(iii)    5C
(c)         5C
(d)         15D

Question 8 (45 Marks)
(a)(i)      5C
(a)(ii)     10C
(a)(iii)    15D
(a)(iv)     5C
(b)         10D

Question 9 (55 Marks)
(a)(i)      10C
(a)(ii)     10C
(b)         10D
(c)         10C
(d)         5C
(e)         10C

## Palette of annotations available to examiners

| Symbol | Name | Meaning in the body of the work | Meaning when used in the right margin |
|---|---|---|---|
| ✔ | Tick | Work of relevance | The work presented in the body of the script merits full credit |
| ✖ | Cross | Incorrect work (distinct from an error) | The work presented in the body of the script merits 0 credit |
| ✳ | Star | Rounding or Unit or Arithmetic error Misreading | |
| 〰 | Horizontal wavy | Error | |
| ✔ₗ | Tick L | | The work presented in the body of the script merits low partial credit |
| ✔ₘ | Tick M | | The work presented in the body of the script merits mid partial credit (or partial credit) |
| ✔ₕ | Tick H | | The work presented in the body of the script merits high partial credit |
| F* | F star | | The work presented in the body of the script merits Full Credit (– 1) |
| [ | Left Bracket | | Another version of this solution is presented elsewhere and it merits equal or higher credit |
| ⸿ | Vertical wavy | No work on this page (portion of the page) | |
| ⊙ | Oversimplify | The candidate has oversimplified the work | |

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**Note:** Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work

e.g. In a **C scale** where ✳ and 〰 and 〰 appear in the body of the work then ✔ₗ should be placed in the right margin.

In the case of a **D scale** with the same level of annotation then ✔ₘ should be placed in the right margin.

A ✔ in the body of the work may sometimes be used to indicate where a portion of the work presented has value and has merited one of the levels of credit described in the marking scheme. The level of credit is them indicated in the right margin.

# Detailed marking notes

## Model Solutions & Marking Notes

**Note:** The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

| Q1 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| **(a)** **(i)** | $f(-3) = 0$ <br><br> $f(-3) = -3^2 + 5(-3) + p = 0$ <br><br> $9 - 15 + p = 0$ <br><br> $p = 6$ <br><br> **Or** <br><br> $x^2 + 5x + p = (x + 3)(x + a)$ <br><br> $\qquad = x^2 + x(a + 3) + 3a$ <br><br> $\qquad a + 3 = 5$ <br><br> $\qquad a = 2$ <br><br> $\qquad p = 3a$ <br><br> $\qquad p = 6$ <br><br> **Or** <br><br> $$\begin{array}{r} x + 2 \phantom{xxxxxx} \\ x + 3 \overline{\smash{\big)}\ x^2 + 5x + p} \\ \underline{x^2 + 2x} \phantom{xxx} \\ 3x + p \\ \underline{3x + 6} \\ p - 6 \end{array}$$ <br> $p - 6 = 0$ <br> $p = 6$ | **Scale 10C (0, 4, 8, 10)** <br> *Low Partial Credit:* <br> Demonstrates understanding of $x + 3$ as factor or $-3$ as root e.g. $(x + 3)$, $f(-3)$ <br><br> *High Partial Credit:* <br> Relevant equation in $p$ (with $p$ as only unknown) |

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| (a)<br>(ii) | $x^2 + 5x + p = (x - \alpha)(x - \alpha - 3)$<br><br>$= x^2 + x(-\alpha - \alpha - 3) + \alpha^2 + 3\alpha$<br><br>$-2\alpha - 3 = 5$<br><br>$\alpha = -4$<br><br>$p = 16 - 12$<br><br>$p = 4$<br><br>**Or**<br><br>$\alpha, \alpha + 3 = $ roots<br><br>$\alpha + \alpha + 3 = -5$<br><br>$2\alpha = -8$<br><br>$\alpha = -4$<br><br>and $\alpha + 3 = -1$<br><br>$p = (-1)(-4) = 4$ | **Scale 5C (0, 3, 4, 5)**<br>*Low Partial Credit:*<br>Demonstrates understanding of 3 as difference of roots e.g. $\alpha$ with $\alpha \pm 3$<br>$x^2 - x(\text{sum}) + \text{product} = 0$<br>One correct value for $p$<br>$x^2 + 5x + p > 0$<br>Sketch of U-shaped quadratic with turning point above the x-axis<br><br>*High Partial Credit:*<br>Relevant equation in $\alpha$ (with $\alpha$ as only unknown)<br><br>*Full Credit (– 1):*<br>$p > 6{\cdot}25$ |
| (a)<br>(iii) | $b^2 - 4ac < 0$<br><br>$5^2 - 4(1)(p) < 0$<br><br>$25 - 4p < 0$<br><br>$4p > 25$<br><br>$p > 6{\cdot}25$<br><br>$p = 7$ and $p = 8$ | **Scale 5C (0, 3, 4, 5)**<br>*Low Partial Credit:*<br>$b^2 - 4ac$<br>One correct value for $p$<br>$x^2 + 5x + p > 0$<br><br>*High Partial Credit:*<br>Relevant inequality in $p$ (with $p$ as only unknown)<br><br>*Full credit (–1):*<br>$p > 6{\cdot}25$ |

**(b)**

$$-1 \leq 2x + 5 \leq 1$$
$$-6 \leq 2x \leq -4$$
$$-3 \leq x \leq -2$$

$$2x + 5 \leq 1$$
$$2x \leq -4$$
$$x \leq -2$$

$$-1 \leq 2x + 5$$
$$-6 \leq 2x$$
$$-3 \leq x$$
$$-3 \leq x \leq -2$$

**Or**

$$(2x + 5)^2 \leq 1$$
$$4x^2 + 20x + 25 \leq 1$$
$$4x^2 + 20x + 24 \leq 0$$
$$x^2 + 5x + 6 \leq 0$$
$$(x + 2)(x + 3) \leq 0$$
$$x = -2, x = -3$$
$$-3 \leq x \leq -2$$

**Scale 5D (0, 2, 3, 4, 5)**
*Low Partial Credit:*
$$(2x + 5)^2 \leq 1$$
one linear inequality

*Mid Partial Credit:*
$$-1 \leq 2x + 5 \leq 1$$

Identifies both linear inequalities
Quadratic inequality involving 0

*High Partial Credit:*
Finding −3 and −2 in Methods 1 or 2
Roots of quadratic found
$-6 \leq 2x \leq -4$ or equivalent

**Note:** Accept   $-3 < x < -2$

| Q2 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| (a) | $$iz_1 = -4 + 3i$$ $$i(iz_1) = i(-4 + 3i)$$ $$-z_1 = -4i + 3i^2$$ $$z_1 = 3 + 4i$$ $$3z_1 - z_2 = 3(3 + 4i) - z_2 = 11 + 17i$$ $$z_2 = 9 + 12i - 11 - 17i$$ $$z_2 = -2 - 5i$$ **Or** $$z_1 = \frac{(-4 + 3i)(-i)}{(i)(-1)}$$ $z_1 = 3 + 4i$ and continues | **Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* Either equation multiplied/divided by $i$ *Mid Partial Credit:* $z_1$ found $z_2$ written in terms of $z_1$ with $z_1$ substituted $z_1$ eliminated *High Partial Credit* $z_1$ found and substituted into second equation $z_2$ found by elimination |
| (b) (i) | $$r = \frac{T_2}{T_1} = \frac{5 - i}{3 + 2i} \times \frac{3 - 2i}{3 - 2i}$$ $$r = \frac{15 - 13i - 2}{9 + 4}$$ $$r = \frac{13 - 13i}{13}$$ $$r = 1 - i$$ | **Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $$\frac{T_2}{T_1}$$ *High Partial Credit:* $$\frac{5 - i}{3 + 2i} \times \frac{3 - 2i}{3 - 2i}$$ |
| (b) (ii) | $$T_9 = ar^8$$ $$T_9 = (3 + 2i)(1 - i)^8$$ $$T_9 = (3 + 2i)\left(\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)\right)^8$$ $$T_9 = (3 + 2i)(\sqrt{2})^8\left(\cos\frac{7\pi(8)}{4} + i\sin\frac{7\pi(8)}{4}\right)$$ $$T_9 = (3 + 2i)(16)(\cos 14\pi + i\sin 14\pi)$$ $$T_9 = (3 + 2i)(16)(1 + 0i)$$ $$T_9 = 48 + 32i$$ | **Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* $T_9 = ar^8$ Any correct use of De Moivre Some use of De Moivre's Theorem on $r$ *Mid Partial Credit:* Modulus and argument found for $r$ *High Partial Credit:* Solution in polar form with some simplification **Note**: Accept candidates $r$ from **(b)(i)** |

| Q3 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| **(a)** | $$fg(x) = f\left(\frac{x+5}{6}\right)$$ $$fg(x) = 6\left(\frac{x+5}{6}\right) - 5 = x$$ $$gf(x) = g(6x - 5)$$ $$gf(x) = \frac{(6x-5)+5}{6} = \frac{6x}{x} = x$$ | **Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $$f\left(\frac{x+5}{6}\right)$$ $$g(6x - 5)$$ Particular case verification <br> *High Partial Credit:* One correct composition simplified to $x$ |
| **(b)** **(i)** | $$\log_5 y = \log_5 5x^2$$ $$\log_5 y = \log_5 5 + \log_5 x^2$$ $$\log_5 y = 1 + 2\log_5 x$$ <br> $a = 1$ and $b = 2$ | **Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $\log_5 5x^2 = \log_5 y$ $\log_5 y = \log_5 5x^2$ <br> *High Partial Credit:* $$\log_5 y = \log_5 5 + \log_5 x^2$$ |
| **(b)** **(ii)** | $$\log_5 y = \log_5 5x^2 = 2 + \log_5\left(\frac{126x}{25} - 1\right)$$ $$\log_5 5x^2 = \log_5\left(\frac{126x}{25} - 1\right) \times 25$$ $$5x^2 = 126x - 25$$ $$5x^2 - 126x + 25 = 0$$ $$(5x - 1)(x - 25) = 0$$ $$x = \frac{1}{5} \text{ or } x = 25$$ $$y = 5x^2 = 5\left(\frac{1}{5}\right)^2 2 = \frac{1}{5}$$ $$\textbf{or } y = 5(25)^2 = 3125$$ | **Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Some relevant use of laws of logs <br> *Mid Partial Credit:* Quadratic equation <br> *High Partial Credit:* $x$ values found <br> **Note**: If 2 is incorrectly (non log) dealt with then award MPC at most <br> **Note**: If incorrect work leads to a non-quadratic equation then award MPC at most |

| Q4 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| **(a)** | $f'(x) = 3x^2 + 2kx + 15$<br><br>$3(3)^2 + 2k(3) + 15 = -12$<br><br>$27 + 6k + 15 = -12$<br><br>$6k = -54$<br><br>$k = -9$ | **Scale 15D (0, 4, 7, 11, 15)**<br>*Low Partial Credit:*<br>Any relevant differentiation<br><br>*Mid Partial Credit:*<br>Expression fully differentiated<br><br>*High Partial Credit:*<br>Derivative fully substituted<br><br>*No Credit:*<br>No differentiation |
| **(b)** | $f'(x) = 3x^2 + 2(-9)x + 15$<br><br>$3x^2 - 18x + 15 = 0$<br><br>$x^2 - 6x + 5 = 0$<br><br>$x = 1 \quad x = 5$<br><br>$f(1) = 15 \qquad (1, 15)$<br>$f(5) = -17 \qquad (5, -17)$<br><br>$m_{g(x)} = -\dfrac{32}{4} = -8$<br><br>$y - 15 = -8(x - 1)$<br><br>$g(x): \quad 8x + y - 23 = 0$ | **Scale 5D (0, 2, 3, 4, 5)**<br>*Low Partial Credit:*<br>Any relevant differentiation<br><br>*Mid Partial Credit:*<br>Both $x$ values found<br><br>*High Partial Credit:*<br>Turning points found |
| **(c)** | $f''(x) = 6x - 18 = 0$<br>$x = 3$<br><br>$f(3) = -1$<br><br>$(3, -1)$ is the point of inflection<br><br>$8(3) + (-1) - 23 = 0$<br><br>$0 = 0$<br><br>$\Rightarrow (3, -1) \in g(x).$ | **Scale 5C (0, 3, 4, 5)**<br>*Low Partial Credit:*<br>$f''(x)$<br><br>*High Partial Credit:*<br>$x$ coordinate of point of inflection found<br>Point of inflection found<br><br>**Note**: Accept candidates $g(x)$ from **(b)** with relevant statement |

| Q5 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| (a) | $$A = \frac{250000(0{\cdot}00287)(1{\cdot}00287)^{300}}{(1{\cdot}00287)^{300} - 1}$$ $$A = €1244{\cdot}06$$ **Or** $$\frac{A}{1{\cdot}00287^1} + \frac{A}{1{\cdot}00287^2} + \cdots \frac{A}{1{\cdot}00287^{300}}$$ $$= 250000$$ $$A\left[\frac{\frac{1}{1{\cdot}00287}\left(\left(\frac{1}{1{\cdot}00287}\right)^{300} - 1\right)}{\frac{1}{1{\cdot}00287} - 1}\right] = 250000$$ $$200{\cdot}9544372 \times A = 250000$$ $$A = €1244{\cdot}06$$ | **Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Formula with some correct substitution $(1{\cdot}00287)$ $300$ *High Partial Credit:* Formula fully substituted |
| (b) | $$\frac{1771}{1{\cdot}003} + \frac{1771}{1{\cdot}003^2} + \cdots + \frac{1771}{1{\cdot}003^{167}}$$ $$+ \frac{1771}{1{\cdot}003^{168}}$$ $$S_{168} = \frac{\frac{1771}{1{\cdot}003}\left[\left(\frac{1}{1{\cdot}003}\right)^{168} - 1\right]}{\frac{1}{1{\cdot}003} - 1}$$ $$= €233438{\cdot}25$$ | **Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* $$\frac{1771}{1{\cdot}003}$$ $168$ *Mid Partial Credit:* $S_{168}$ formula with some substitution *High Partial Credit:* Formula fully substituted |

| Q6 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| **(a)** | $$f(x) = (3x - 5)(2x + 4)$$ $$= 6x^2 + 2x - 20$$ $$f(x + h) = 6(x + h)^2 + 2(x + h) - 20$$ $$= 6x^2 + 12hx + 6h^2 + 2x + 2h - 20$$ $$f(x + h) - f(x) = 12hx + 6h^2 + 2h$$ $$\frac{f(x + h) - f(x)}{h} = 12x + 6h + 2$$ $$\lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = 12x + 2$$ $$f'(x) = 12x + 2$$ | **Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Some substitution into $f(x + h)$ or $y + \Delta y$ *High Partial Credit:* $$f(x + h) - f(x) = 12hx + 6h^2 + 2h$$ *No Credit*: Not from first principles $(3x - 5)(2x + 4) = 6x^2 + 2x - 20$ |
| **(b) (i)** | $$h'(x) = \frac{1}{2}\left(\frac{1}{2x + 3}\right)(2)$$ $$= \frac{1}{2x + 3}$$ | **Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Any relevant differentiation *High Partial Credit:* $$\frac{1}{2}\left(\frac{1}{2x + 3}\right)$$ |
| **(b) (ii)** | $$\int_0^A \frac{1}{2x + 3}\,dx = ln3$$ $$\frac{1}{2}\ln(2x + 3) \ \Big|_0^A = \ln 3$$ $$\frac{1}{2}\left(\ln(2A + 3) - \ln 3\right) = \ln 3$$ $$\frac{1}{2}\ln\left(\frac{2A + 3}{3}\right) = \ln 3$$ $$\ln\left(\frac{2A + 3}{3}\right)^{\frac{1}{2}} = \ln 3$$ $$\left(\frac{2A + 3}{3}\right)^{\frac{1}{2}} = 3$$ $$\frac{2A + 3}{3} = 9$$ $$2A + 3 = 27$$ $$2A = 24$$ $$A = 12$$ | **Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* Integration indicated *Mid Partial Credit:* $\frac{1}{2}\ln(2x + 3) \ \Big|_0^A$ Substitutes limits into integral and stops Correct integration with some substitution *High Partial Credit:* Integral evaluated at $x = A$ only (i.e. omits ln 3 on LHS and finishes **Note**: Must have integration to gain any credit |

| Q7 | Model Solution – 50 Marks | Marking Notes |
|---|---|---|
| **(a) (i)** | | **Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* One correct new entry <br><br> *High Partial Credit:* Three correct new entries |

| T. | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ | $T_6$ | $T_7$ | $T_8$ |
|---|---|---|---|---|---|---|---|---|
| No. | 1 | 3 | 6 | 10 | 15 | 21 | 28 | 36 |

**(a) (ii)**

$$\frac{n}{2}(n + 1) = 1275$$

$$n^2 + n - 2250 = 0$$

$$(n - 50)(n + 51)$$

$$n = 50$$

1275 is the 50th triangular number

**Scale 5C (0, 3, 4, 5)**
*Low Partial Credit:*
$$\frac{n}{2}(n + 1) = 1275$$

*High Partial Credit:*
$$n = 50$$

Note: accept $T_{50}$ as valid reason

**(b) (i)**

$$T_{n+1} = T_n + (n + 1)$$

$$= \frac{n}{2}(n + 1) + (n + 1)$$

$$= \frac{n(n + 1) + 2(n + 1)}{2}$$

$$= \frac{(n + 1)(n + 2)}{2}$$

**Scale 5C (0, 3, 4, 5)**
*Low Partial Credit:*
2 identified as C.D.
Correct numerator

*High Partial Credit:*
$$= \frac{n(n + 1) + 2(n + 1)}{2}$$

**(b) (ii)**

$$T_{n+1} + T_n$$

$$= \frac{(n + 1)(n + 2)}{2} + \frac{n}{2}(n + 1)$$

$$= \frac{(n + 1)(2n + 2)}{2}$$

$$= \frac{2(n + 1)(n + 1)}{2}$$

$$= (n + 1)^2$$

**Scale 5C (0, 3, 4, 5)**
*Low Partial Credit:*
$T_{n+1} + T_n$ with some substitution
Particular case verification

*High Partial Credit:*
$$\frac{(n + 1)(n + 2)}{2} + \frac{n}{2}(n + 1)$$

| | | |
|---|---|---|
| **(b)**<br>**(iii)** | $(n+1)^2 = 12544$<br><br>$n+1 = \sqrt{12544} = 112$<br><br>$n = 111$<br><br>$n = 111$<br><br>$T_{111}$ is the smaller term<br><br>$T_{111} = \dfrac{111(112)}{2}$<br><br>$T_{111} = 6216$ | **Scale 5C (0, 3, 4, 5)**<br>*Low Partial Credit:*<br>$(n+1)^2$<br><br>*High Partial Credit:*<br>$n = 111$, or $n = 112$ |
| **(c)** | $N_3 = \left( \dfrac{\left(3 + 2\sqrt{2}\right)^3 - \left(3 - 2\sqrt{2}\right)^3}{4\sqrt{2}} \right)^2$<br><br><br>$= 1225$ | **Scale 5C (0, 3, 4, 5)**<br>*Low Partial Credit:*<br>Formula with some substitution<br><br>*High Partial Credit:*<br>Formula fully substituted<br><br>*Full Credit:*<br>Correct answer with no work shown |

**(d)**

$$1^2 + 2^2 + 3^2 + \cdots + n^2 =$$

$$\frac{n(n+1)(2n+1)}{6}$$

$$\boldsymbol{P(1)}: \ 1 = \frac{1(2)(3)}{6}$$

$$\boldsymbol{P(k)}: 1 + 4 + 9 + \cdots + k^2 =$$

$$\frac{k(k+1)(2k+1)}{6}$$

$$\boldsymbol{P(k+1)}: 1 + 4 + 9 + \cdots + k^2 + (k+1)^2$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$LHS = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$LHS = \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$LHS = \frac{(k+1)[k(2k+1) + 6(k+1)]}{6}$$

$$LHS = \frac{(k+1)[2k^2 + 7k + 6]}{6}$$

$$\frac{(k+1)(k+2)(2k+3)}{6} = RHS$$

Thus the proposition is true for $n = k + 1$ provided it is true for $n = k$ but it is true for $n = 1$ and therefore true for all positive integers.

**Scale 15D (0, 4, 7, 11, 15)**

*Low Partial Credit:*
  Step $P(1)$

*Mid Partial Credit:*
  Step $P(k+1)$

*High Partial Credit:*
  Uses *Step* $P(k)$ to prove *Step* $P(k+1)$

*Full Credit(–1):*
Concluding statement missing

  **Note:** Accept *Step* $P(1)$, *Step* $P(k)$,
    *Step* $P(k+1)$ in any order

| Q8 | Model Solution – 45 Marks | Marking Notes |
|---|---|---|
| (a)<br>(i) | $$\cos\theta = \frac{x}{5} \qquad \sin\theta = \frac{y}{5}$$ $$5\cos\theta = x \qquad 5\sin\theta = y$$ $$(x,y) = (5\cos\theta,\ 5\sin\theta)$$ $$\therefore a = 5,\ \ b = 5$$ | **Scale 5C (0, 3, 4, 5)** <br> *Low Partial Credit:* <br> $\cos\theta = \frac{x}{5}$ or equivalent <br><br> *High Partial Credit:* <br> $a$ or $b$ found <br> Correct answer without work |
| (a)<br>(ii) | $$A(\theta) = (10\cos\theta)\times(10\sin\theta)$$ $$A(\theta) = 100\cos\theta\sin\theta$$ $$= 50\times 2\cos\theta\sin\theta$$ $$= 50(\sin 2\theta)$$ | **Scale 10C (0, 4, 8, 10)** <br> *Low Partial Credit:* <br> $\quad xy$ <br> $\quad (10\cos\theta)\times(10\sin\theta)$ <br><br> *High Partial Credit:* <br> $\quad 100\cos\theta\sin\theta$ |
| (a)<br>(iii) | $$A(\theta) = 50\sin 2\theta$$ $$A'(\theta) = 50\cos 2\theta \times 2$$ $$A'(\theta) = 100\cos 2\theta = 0$$ $$\cos 2\theta = 0$$ $$2\theta = \frac{\pi}{2}$$ $$\theta = \frac{\pi}{4}$$ $$2x = 2\left(5\cos\left(\frac{\pi}{4}\right)\right) = 5\sqrt{2}$$ $$2y = 2\left(5\sin\left(\frac{\pi}{4}\right)\right) = 5\sqrt{2}$$ $$\Longrightarrow \text{Square}$$ | **Scale 15D (0, 4, 7, 11, 15)** <br> *Low Partial Credit:* <br> $a'(\theta)$ <br> States $\frac{dy}{dx} = 0$ <br><br> *Mid Partial Credit:* <br> Correct differentiation <br><br> *High Partial Credit:* <br> Value of $\theta$ at maximum found <br> Value of $x$ or $y$ at maximum fully substituted <br><br> *No Credit:* <br> No differentiation |
| (a)<br>(iv) | Max area $= 5\sqrt{2}\times 5\sqrt{2}$ $$= 50 \text{ Square units}$$ **Or** Max area $= 50(\sin 2\theta)$ $$50(\sin\frac{\pi}{2})$$ $$= 50 \text{ Square units}$$ | **Scale 5C (0, 3, 4, 5)** <br> *Low Partial Credit:* <br> $xy$ <br> length $\times$ width <br> $50(\sin 2\theta)$ <br><br> *High Partial Credit:* <br> Area formula fully substituted |

**(b)**

$$\frac{dx}{dt} = \frac{dx}{dl} \cdot \frac{dl}{dt}$$

$$\frac{2}{5} = \frac{x}{l+x}$$

$$2l + 2x = 5x$$

$$x = \frac{2}{3}l$$

$$\frac{dx}{dl} = \frac{2}{3}$$

$$\frac{dx}{dt} = \frac{2}{3} \times \frac{3}{2}$$

$$\frac{dx}{dt} = 1 \text{ m/sec}$$

**Scale 10D (0, 3, 5, 8, 10)**

*Low Partial Credit:*

$\frac{dx}{dt}$ or $\frac{dx}{dl}$ or $\frac{dl}{dt}$ given

Reference to similar triangles

$\frac{2}{5}$ or $\frac{5}{2}$

*Mid Partial Credit:*

$\frac{dx}{dt} = \frac{dx}{dl} \cdot \frac{dl}{dt}$ or equivalent with one relevant substitution

$x = \frac{2}{3}l$

*High Partial Credit:*

$\frac{dx}{dl}$ and $\frac{dl}{dt}$ found

20

| Q9 | Model Solution – 55 Marks | Marking Notes |
|---|---|---|
| (a) (i) | $N(t) = 450e^{0.065t}$<br><br>$N(4.5) = 450e^{0.065(4.5)}$<br><br>$= 602.89$<br><br>$= 603$ | **Scale 10C (0, 4, 8, 10)**<br>*Low Partial Credit:*<br>Some substitution into function<br>Correct answer without work<br><br>*High Partial Credit:*<br>$450e^{0.065(4.5)}$ |
| (a) (ii) | $N(t) = 450e^{0.065t}$<br><br>$\dfrac{N(t)}{450} = e^{0.065t}$<br><br>Convert to log equation<br><br>$\ln\left(\dfrac{N(t)}{450}\right) = 0.065t$<br><br>$\ln\bigl(N(t)\bigr) - \ln 450 = 0.065t$<br><br>$\dfrac{\ln\bigl(N(t)\bigr) - \ln 450}{0.065} = t$<br><br>$t = \dfrac{\ln(790) - \ln 450}{0.065}$<br><br>$t = 8.7$ | **Scale 10C (0, 4, 8, 10)**<br>*Low Partial Credit:*<br>Some substitution into function<br>Full substitution and stops<br><br>*High Partial Credit:*<br>Equation in $t$ ( i.e. logs handled correctly) |

| | | |
|---|---|---|
| **(b)** | $$\frac{1}{9}\int_{3}^{12} 450e^{0.065t}\,dt$$ $$= \frac{450}{9(0.065)}\left[e^{12(.065)} - e^{3(0.065)}\right]$$ $$= 743.2$$ Average no. $= 743$ | **Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Integration indicated *Mid Partial Credit:* Integration correct *High Partial Credit:* Substitutes limits into integral and stops **Note**: Must have integration to gain any credit |
| **(c)** | $$N'(t) = 450e^{0.065t} \times 0.065$$ $$N'(t) = 29.25e^{0.065t}$$ $$N'(12) = 29.25e^{0.065(12)}$$ $$= 63.8$$ At hour 12 the population is growing at a rate of 64 bacteria per hour **or** At hour 12 the population is growing at a rate of 63.8 bacteria per hour | **Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $N'(t)$ stated or indicated *High Partial Credit:* Derivative fully substituted $N'(12) = 63.8$ and stops |

| | | |
|---|---|---|
| **(d)** | $N'(t) = 29 \cdot 25 e^{0 \cdot 065k} > 90$ <br><br> $e^{0 \cdot 065k} > 29 \cdot 25$ <br><br> $k > \dfrac{\ln \dfrac{90}{29 \cdot 25}}{0 \cdot 065}$ <br><br> $k > 17 \cdot 29$ <br><br> $k = 18$ | **Scale 10C (0, 4, 8, 10)** <br> *Low Partial Credit:* <br> $\quad 29 \cdot 25 e^{0 \cdot 065k} > 90$ <br><br> *High Partial Credit:* <br> Equation in $k$ ( i.e. taking logs handled correctly) <br><br> *No Credit:* <br> No differentiation <br><br> **Note:** if $k > 17 \cdot 29 \Rightarrow k = 17$ <br> Award Full credit (–1) |
| **(e)** | $450 e^{0 \cdot 065t} = 220 e^{0 \cdot 17t}$ <br><br> $\dfrac{450}{220} = \dfrac{e^{0 \cdot 17t}}{e^{0 \cdot 065t}}$ <br><br> $\dfrac{450}{220} = e^{0 \cdot 105t}$ <br><br> $\ln \left( \dfrac{450}{220} \right) = 0 \cdot 105t$ <br><br> $\dfrac{\ln \left( \dfrac{450}{220} \right)}{0 \cdot 105} = t$ <br><br> $t = 6 \cdot 82$ <br><br> $t = 7 \text{ hours}$ | **Scale 10C (0, 4, 8, 10)** <br> *Low Partial Credit:* <br> $\quad 450 e^{0 \cdot 065t} = 220 e^{0 \cdot 17t}$ <br><br> *High Partial Credit:* <br> Equation in $t$ ( i.e. taking logs handled correctly) |

Leaving Certificate 2020

Marking Scheme

# Mathematics

Higher Level

Paper 2

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# Marking Scheme – Paper 2, Section A and Section B

## Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated.  Scales labelled A divide candidate responses into two categories (correct and incorrect).  Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on.  The scales and the marks that they generate are summarised in this table:

| Scale label | A | B | C | D | E |
|---|---|---|---|---|---|
| No of categories | 2 | 3 | 4 | 5 | 6 |
| 5 mark scales | | 0, 2, 5 | 0, 3, 4, 5 | 0, 2, 3, 4, 5 | |
| 10 mark scales | | | 0, 4, 8, 10 | 0, 3, 5, 8, 10 | |
| 15 mark scales | | | 0, 5, 10, 15 | 0, 4, 7, 11, 15 | |
| 20 mark scales | | | | | |
| 25 mark scales | | | | | |

A general descriptor of each point on each scale is given below.  More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

## Marking scales – level descriptors

### A-scales (two categories)
- incorrect response
- correct response

### B-scales (three categories)
- response of no substantial merit
- partially correct response
- correct response

### C-scales (four categories)
- response of no substantial merit
- response with some merit
- almost correct response
- correct response

### D-scales (five categories)
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

### E-scales (six categories)
- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

**NOTE:** In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded.
Rounding and units penalty to be applied only once in each section (a), (b), (c) etc.
Throughout the scheme indicate by use of * where an arithmetic error occurs.

### Summary of mark allocations and scales to be applied

| *Section A* | | | *Section B* | | |
|---|---|---|---|---|---|
| Question 1 | | | Question 7 | | |
| (a) | 15D | | (a)(i) | 10C | |
| (b) | 10D | | (a)(ii) | 10C | |
| | | | (a)(iii) | 10C | |
| Question 2 | | | (b) | 10D | |
| (a) | 10D | | (c) | 5D | |
| (b) | 15D | | (d) | 10D | |
| | | | | | |
| Question 3 | | | Question 8 | | |
| (a) | 15C | | (a)(i) | 15D | |
| (b) | 10D | | (a)(ii) | 10D | |
| | | | (b)(i) | 5B | |
| Question 4 | | | (b)(ii) | 10D | |
| (a) | 10C | | (c) | 10C | |
| (b) | 15D | | (d) | 10D | |
| | | | (e) | 10D | |
| Question 5 | | | | | |
| (a)(i) | 15C | | Question 9 | | |
| (a)(ii) | 5C | | (a) | 15C | |
| (b) | 5C | | (b) | 5C | |
| | | | (c) | 5D | |
| Question 6 | | | | | |
| (a) | 10D | | | | |
| (b)(i) | 10C | | | | |
| (b)(ii) | 5C | | | | |

## Model Solutions & Marking Notes

**Note**: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions.  Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

| Q1 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| (a) | Slope of $BC$ $m = \frac{3+12}{-4-6} = -\frac{3}{2}$ <br><br> Equation $BC$ $\quad 3x + 2y + 6 = 0.$ <br><br> Perp. Distance from $A$ to line $BC$ <br><br> $\frac{3(2)+2(-6)+6}{\sqrt{3^2+2^2}} = \frac{6-12+6}{\sqrt{13}} = \frac{0}{\sqrt{13}} = 0.$ <br><br><br> Therefore $A$, $B$ and $C$ are collinear. | **Scale 15D (0, 4, 7, 11, 15)** <br> *Low Partial Credit:* <br>   Slope formula with some substitution <br>   Equation of line formula with some substitution <br>   Effort at finding are of triangle $ABC$ <br><br> *Mid Partial Credit:* <br> Equation of $BC$ <br><br> *High Partial Credit:* <br> Perp. Distance formula with some substitution from relevant line <br> Area of triangle $ABC = 0$ but perp. distance not explicit <br><br> *Full credit* $(-1)$ <br> Distance $= 0$ but conclusion omitted <br> Area of triangle $ABC = 0$ **and** perp. dist. $= 0$ but conclusion omitted |

**(b)**

$$\text{Slope of } a = \frac{1}{2}$$

$$\text{Slope of } b = \tan 60° = \sqrt{3}$$

$$\tan\theta = \pm\frac{\sqrt{3} - \frac{1}{2}}{1 + \frac{\sqrt{3}}{2}} = \pm\frac{2\sqrt{3} - 1}{2 + \sqrt{3}}$$

$$= \pm\frac{(2\sqrt{3} - 1)(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})}$$

$$= \pm(-8 + 5\sqrt{3})$$

$$\theta = \tan^{-1}(-8 + 5\sqrt{3})$$

$$\theta = 33{\cdot}435°$$

**Or**

$$\theta + \tan^{-1}\frac{1}{2} + 120° = 180°`$$

$$\theta + 26{\cdot}565° + 120° = 180°$$

$$\theta = 33{\cdot}435°$$

**Scale 10D (0, 3, 5, 8, 10)**
*Low Partial Credit:*
Slope of $a = \frac{1}{2}$
Slope of $b = \tan 60°$

*Mid Partial Credit:*
Tan formula with some relevant substitution

*High Partial Credit:*
Tan formula fully substituted

*Full credit* (–1)
$$\theta = +\tan^{-1}(-8 + 5\sqrt{3})$$

**Scale 10D (0, 3, 5, 8, 10)**
*Low Partial Credit:*
Slope of $a = \frac{1}{2}$
120°

*Mid Partial Credit:*
$$\tan^{-1}\frac{1}{2} + 120°$$

*High Partial Credit:*
$\theta + 26{\cdot}565° + 120° = 180°$
and fails to finish

| Q2 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| **(a)** | Centre: $(2, -1)$<br><br>Radius: $\sqrt{2^2 + (-1)^2 + 4} = 3$<br><br>Distance from centre to B: $\sqrt{90}$<br><br>Pythagoras: $\|BT\|^2 = 90 - 3^2 = 81$<br><br>$\qquad \Longrightarrow \|BT\| = 9$ | **Scale 10D (0, 3, 5, 8, 10)**<br>*Low Partial Credit:*<br>Centre or radius<br><br>*Mid Partial Credit:*<br>$\sqrt{90}$<br><br>*High Partial Credit:*<br>Pythagoras fully substituted (: $\|BT\|^2$) |
| **(b)** | Centre $(-g, 0)$.<br><br>Radius = $\sqrt{g^2 + (0)^2 - c} = 5$<br><br>$\Longrightarrow g^2 - c = 25$   Equation (i)<br><br>Equation is $x^2 + y^2 + 2gx + c = 0$<br><br>Sub (1, 4):<br><br>$1^2 + 4^2 + 2g(1) + c = 0$<br><br>$\Longrightarrow 17 + 2g + c = 0$   Equation (ii)<br><br>Solve (i) and (ii)<br><br>$17 + 2g + (g^2 - 25) = 0$<br><br>$\qquad \Longrightarrow g^2 + 2g - 8 = 0$<br><br>Solve for g:<br><br>$g = 2$ and $g = -4$<br><br>Centres are $(-2, 0)$ and $(4, 0)$<br><br>Equations:<br><br>$(x + 2)^2 + y^2 = 25$,<br><br>$\quad (x - 4)^2 + y^2 = 25$<br><br>**Or** | **Scale 15D (0, 4, 7, 11, 15)**<br>*Low Partial Credit:*<br>Centre $(-g, 0)$ or equivalent<br>Some substitution of (1, 4) into general equation of circle<br><br>*Mid Partial Credit:*<br>2 relevant equations in $g$ and $c$<br><br>*High Partial Credit:*<br>Quadratic in $g$ ($g^2 + 2g - 8 = 0$ or equivalent) |

Centre: $(-g, 0)$

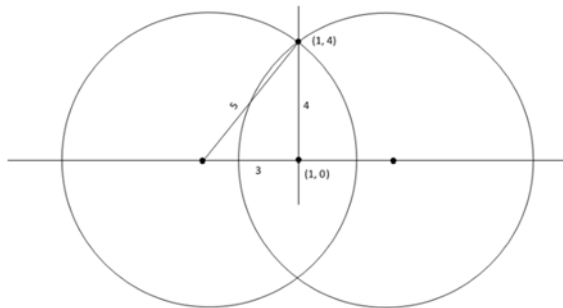$$\sqrt{(1+g)^2 + (4-0)^2} = 5$$
$$(1+g)^2 = 9$$
$$1 + g = \pm 3$$

$g = -4$ or $g = 2$

Equations:

$(x+2)^2 + y^2 = 25,$

$\quad (x-4)^2 + y^2 = 25$

**or**



Centres $(-2, 0)$ and $(4, 0)$; radius $= 5$

Equations:

$(x+2)^2 + y^2 = 25,$

$\quad (x-4)^2 + y^2 = 25$

**Scale 15D (0, 4, 7, 11, 15)**
*Low Partial Credit:*
Centre $(-g, 0)$ or equivalent
Some substitution into distance formula

*Mid Partial Credit:*
Distance formula fully substituted

*High Partial Credit:*
Quadratic in $g$

**Scale 15D (0, 4, 7, 11, 15)**
*Low Partial Credit:*
Diagram with (1, 0) identified
*Mid Partial Credit:*
-2 or 4 identified

*High Partial Credit:*
$g = -4$ and $g = 2$

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| Q3 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| **(a)** | $$\frac{6}{\sin 17°} = \frac{|HF|}{\sin 35°}$$ $$|HF| = \frac{6 \sin 35°}{\sin 17°} = 11 \cdot 77$$ $$\frac{11 \cdot 77}{\sin 95°} = \frac{x}{\sin 33°}$$ $$x = \frac{11 \cdot 77 (\sin 33°)}{\sin 95°}$$ $$x = 6 \cdot 43 \text{ m}$$ | **Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* $\lvert\angle FHE\rvert = 17°$ $\lvert\angle GHF\rvert = 33°$ Some relevant substitution into relevant formula  *High Partial Credit:* $\lvert HF\rvert$ found and stops $\lvert HE\rvert = 16 \cdot 17$ found and stops Incorrect value of $\lvert HF\rvert$ (or $\lvert HE\rvert$ ) used correctly to find $x$ |
| **(b)** | $$\lvert\angle BOA\rvert = 60° \implies \lvert\angle COA\rvert = 30°$$ $$\sin\angle COA = \frac{r}{DO} = \frac{1}{2}$$ $$\implies \lvert DO\rvert = 2r$$ $$\implies \lvert OC\rvert = 3r$$ $$\text{Area } c = \pi r^2$$ $$\text{Area } s = \pi (3r)^2 = 9\pi r^2$$ $$\text{Area } s : \text{Area } c = 9 : 1 \implies k = 9$$ | **Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* $30°$ Area $c = \pi r^2$  *Mid Partial Credit:* $\lvert DO\rvert = 2r$  *High Partial Credit:* $\lvert OC\rvert = 3r$ |

| Q4 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| **(a)** | Reference angle: $\dfrac{\pi}{6}$ <br><br> $2^{\text{nd}}$ Quadrant: $\pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$ <br><br> $\dfrac{\theta}{2} = \dfrac{5\pi}{6} + 2n\pi$ <br><br> $\theta = \dfrac{5\pi}{3} + 4n\pi$ <br><br> $n = 0 \implies \theta = \dfrac{5\pi}{3} = 300°$ <br><br> $4^{\text{nd}}$ Quadrant: $2\pi - \dfrac{\pi}{6} = \dfrac{11\pi}{6}$ <br><br> $\dfrac{\theta}{2} = \dfrac{11\pi}{6} + 2n\pi$ <br><br> $\theta = \dfrac{11\pi}{3} + 4n\pi$ <br><br> $n = 0 \implies \theta = \dfrac{11\pi}{3} = 660°$ | **Scale 15D (0, 4, 7, 11, 15)** <br> *Low Partial Credit:* <br> 30° or –30° <br> Mention of $2^{\text{nd}}$ or $4^{\text{th}}$ quadrants <br><br> *Mid Partial Credit* <br> 150° or 330° or equivalent <br><br> *High Partial Credit:* <br> 150° and 330° or equivalent |
| **(b)** | Area of $\Delta COA$ = Area of Sector $-\ 21$ <br><br> $= \dfrac{1}{2}r^2\theta - 21 = 8{\cdot}4$ <br><br> Area of $\Delta COA$:  $\dfrac{1}{2}|CO||7|\sin 1{\cdot}2 = 8{\cdot}4$ <br><br> $|CO| = \dfrac{8{\cdot}4}{3{\cdot}5\sin 1{\cdot}2} = 2{\cdot}57$ <br><br> $|BC| = 7 - 2{\cdot}6 = 4{\cdot}4$ cm | **Scale 10D (0, 3, 5, 8, 10)** <br> *Low Partial Credit:* <br> Area of $\Delta COA$ <br> Area of Sector $COA$ <br><br> *Mid Partial Credit:* <br> Area of $\Delta COA$ = Area of Sector $-\ 21$ <br><br> *High Partial Credit:* <br> $\dfrac{1}{2}|CO||7|\sin 1{\cdot}2 = 8{\cdot}4$ <br><br> *Full credit* $(-1)$ <br> Distance $|CO|$ found and stops |

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| Q5 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| (a) (i) | $$P(B|A) = \frac{P(A \cap B)}{P(A)}$$ $$P(B|A) = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$ | **Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Formula for $P(B|A)$  *High Partial Credit:* Formula fully substituted |
| (a) (ii) | $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$ $$\frac{11}{12} = \frac{3}{4} + P(B) - \frac{1}{2}$$ $$\frac{11}{12} - \frac{1}{4} = P(B) = \frac{2}{3}$$ Check if:  $P(A) \times P(B) = P(A \cap B)$ $$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2} = P(A \cap B)$$ $\Rightarrow$ Independent **or** $$P(B|A) = P(B)$$ $$\frac{2}{3} = \frac{2}{3}$$ $\Rightarrow$ Independent | **Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Condition for independent events  *High Partial Credit:* $P(B) = \frac{2}{3}$ $P(A) \times P(B) = P(A \cap B)$ fully checked for any relevant value (< 1) of $P(B)$ with a valid conclusion |

**(b)**

| Add | 1 | 1 | 2 | 3 |
|-----|---|---|---|---|
| **1** | 2 | 2 | 3 | 4 |
| **1** | 2 | 2 | 3 | 4 |
| **2** | 3 | 3 | 4 | 5 |
| **3** | 4 | 4 | 5 | 6 |

| Rem. | 1 | 1 | 2 | 3 |
|------|---|---|---|---|
| **1** | 2 | 2 | 0 | 1 |
| **1** | 2 | 2 | 0 | 1 |
| **2** | 0 | 0 | 1 | 2 |
| **3** | 1 | 1 | 2 | 0 |

Lee has 6 chances to win.
The others only have 5 chances
$\Rightarrow$ It is not a fair game

**Scale 5C (0, 3, 4, 5)**
*Low Partial Credit:*
  Any relevant listing of remainders/sums

*High Partial Credit:*
  All remainders listed but no conclusion or incorrect conclusion or unsound conclusion

| Q6 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| **(a)** | | |

|  | D | P | H/E |
|---|---|---|---|
|  | 0·3 | 0·6 | 0·1 |
|  | × 0·7 | × 0·25 | × 0·09 |
| **VW** | 0·21 | 0·15 | 0·009 |

$$P(VW) = 0{\cdot}21 + 0{\cdot}15 + 0{\cdot}009$$
$$= 0{\cdot}369$$

**Scale 10D (0, 3, 5, 8, 10)**
*Low Partial Credit:*
 Any relevant probability from line 1 written

*Mid Partial Credit:*
 Any 1 relevant probability from line 3 formulated or written

*High Partial Credit:*
 All 3 relevant probability from line 3 formulated or written

---

**(b) (i)**

$$\binom{5}{2}\left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^3 \frac{1}{4} = \frac{135}{2048}$$

**Scale 10C (0, 4, 8, 10)**
*Low Partial Credit:*
$\binom{5}{2}$ or $\frac{3}{4}$ or $\left(\frac{1}{4}\right)^2$ or $\left(\frac{1}{4}\right)^3$

*High Partial Credit:*
$\binom{5}{2}\left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^y$ where $x, y \neq 1$

---

**(b) (ii)**

P(2 or less) = P(0 pass + 1 pass + 2 pass)

$$P(0\ \text{pass}) = \left(\frac{1}{2}\right)^n$$

$$P(1\ \text{pass}) = \left[\binom{n}{1}\left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{n-1}\right]$$

$$P(2\ \text{pass}) = \left[\binom{n}{2}\left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{n-2}\right]$$

$$P(\leq 2) = \frac{1}{2^n} + \left[\frac{n}{2^n}\right] + \left[\frac{n(n-1)}{2^{n+1}}\right]$$

$$= \frac{2 + 2n + n^2 - n}{2^{n+1}} = \frac{n^2 + n + 2}{2^{n+1}}$$

$$\implies a = 1,\ \ b = 1,\ \ c = 2.$$

**Scale 5C (0, 3, 4, 5)**
*Low Partial Credit:*
P(0 pass + 1 pass + 2 pass)

*High Partial Credit:*
Any two of $\left(\frac{1}{2}\right)^n$ or $\left[\binom{n}{1}\left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{n-1}\right]$ or
$\left[\binom{n}{2}\left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{n-2}\right]$

| Q7 | Model Solution – 55 Marks | Marking Notes |
|---|---|---|
| **(a)**<br>**(i)** | $$9^2 = 3 \cdot 3^2 + h^2$$ $$h^2 = 81 - 10 \cdot 89$$ $$h = 8 \cdot 37$$ | **Scale 10C (0, 4, 8, 10)**<br>*Low Partial Credit:*<br>Pythagoras formulated<br><br>*High Partial Credit:*<br>$\sqrt{9^2 - 3 \cdot 3^2}$ or equivalent |
| **(a)**<br>**(ii)** | $$CSA = \pi r l = \pi 3 \cdot 3(9) = 93 \cdot 31 \text{ cm}^2$$ | **Scale 10C (0, 4, 8, 10)**<br>*Low Partial Credit:*<br>Formula for **CSA** with some substitution<br><br>*High Partial Credit:*<br>Formula fully substituted |
| **(a)**<br>**(iii)** | Circumference of cup $= 2\pi r = 2\pi(3 \cdot 3)$<br>Arc length of sector $= \dfrac{2\pi \times 9\theta}{360°}$<br><br>$$2\pi(3 \cdot 3) = \dfrac{2\pi \times 9\theta}{360°}$$ $$\theta = \dfrac{3 \cdot 3(360)}{9} = 132°$$ | **Scale 10C (0, 4, 8, 10)**<br>*Low Partial Credit:*<br>Formula for circumference **or** arc length with some substitution<br><br>*High Partial Credit:*<br>Both formulas fully substituted |
| **(b)** | $$\dfrac{3 \cdot 3}{8 \cdot 37} = \dfrac{r}{7 \cdot 37}$$ $$r = 2 \cdot 905 \text{ cm}$$ $$v = \dfrac{1}{3}\pi(2 \cdot 905)^2 7 \cdot 37$$ $$65 \cdot 16 \text{ cm}^3$$ $$65 \cdot 2 \text{ cm}^3$$ | **Scale 10D (0, 3, 5, 8, 10)**<br>*Low Partial Credit:*<br>Any relevant effort to find $r$ using similar triangles<br><br>*Mid Partial Credit:*<br>$r$ found<br><br>*High Partial Credit:*<br>Volume formula fully substituted<br><br>Note: If $r = 3 \cdot 3$ used then award MPC at most |

| | | |
|---|---|---|
| **(c)** | Volume of water in one second $\pi 0 \cdot 8^2 (2 \cdot 5)$<br><br>$= 5 \cdot 0265 \text{ cm}^3$<br><br>Time taken is $\dfrac{65 \cdot 2}{\pi 0 \cdot 8^2 (2 \cdot 5)} = 13$ | **Scale 5D (0, 2, 3, 4, 5)**<br>*Low Partial Credit:*<br>  Any relevant effort to find volume of water<br><br>*Mid Partial Credit:*<br>  $\pi 0 \cdot 8^2 (2 \cdot 5)$<br><br>*High Partial Credit:*<br>  Time formula fully substituted<br><br>**Note**: Accept work using candidates volume from part (b) |
| **(d)** | $$\dfrac{3 \cdot 3}{8 \cdot 37} = \dfrac{r}{h}$$<br>$$r = \dfrac{3 \cdot 3 h}{8 \cdot 37}$$<br>$$v = \dfrac{1}{3} \pi \left(\dfrac{3 \cdot 3 h}{8 \cdot 37}\right)^2 h = 60$$<br>$$h^3 = \dfrac{60 \times 8 \cdot 37^2 \times 3}{\pi 3 \cdot 3^2}$$<br>$$h = \sqrt[3]{\dfrac{60 \times 8 \cdot 37^2 \times 3}{\pi 3 \cdot 3^2}} = 7 \cdot 169$$<br>$$x = 8 \cdot 37 - 7 \cdot 169 = 1 \cdot 2$$ | **Scale 10D (0, 3, 5, 8, 10)**<br>*Low Partial Credit:*<br>Effort to link $r$ and $h$<br><br>*Mid Partial Credit*<br>$r$ and $h$ linked<br><br>*High Partial Credit:*<br>$h^3 = \dfrac{60 \times 8 \cdot 37^2 \times 3}{\pi 3 \cdot 3^2}$  or equivalent |

| Q8 | Model Solution – 70 Marks | Marking Notes |
|---|---|---|
| **(a)**<br>**(i)** | $$z = \frac{x - \bar{x}}{\sigma}$$ $$\frac{x - 280}{90} = 0{\cdot}68$$ $$\Rightarrow x = 341{\cdot}2$$ $$x = 342$$ | **Scale 15D(0, 4, 7, 11, 15)**<br>*Low Partial Credit:*<br>$\mu$ or $\sigma$ identified<br><br>*Mid Partial Credit:*<br>$\quad 0{\cdot}68$<br><br>*High Partial Credit:*<br>Equation in $x$ fully substituted and stops or<br>$\quad$ continues incorrectly |
| **(a)**<br>**(ii)** | Eileen's z-score $= \frac{260-280}{90} = -0{\cdot}222 = z$<br><br>40% z-score $= -0{\cdot}25$ i.e. z score for 60%<br><br>$$-0{\cdot}222 > -0{\cdot}25$$<br>Eileen is eligible to re-sit the test.<br><br>or<br><br>$$P(0{\cdot}222) = 0{\cdot}5871$$ $$1 - 0{\cdot}5871 = 0{\cdot}4129$$ $$41{\cdot}29\%$$ | **Scale 10D(0, 3, 5, 8, 10)**<br>*Low Partial Credit:*<br>$\mu$ or $\sigma$ identified<br><br>*Mid Partial Credit:*<br>$\frac{260-280}{90}$ or $-0{\cdot}222$ or $-0{\cdot}25$<br><br>*High Partial Credit:*<br>$\quad -0{\cdot}222$ and $-0{\cdot}25$<br><br>Note: Allow $-0{\cdot}26$ |
| **(b)**<br>**(i)** | 95% of the of the data lies in the interval<br>$-1{\cdot}96 \leq z \leq 1{\cdot}96$ | **Scale 5B (0, 2, 5)**<br>*Partial Credit:*<br>95% without context |

| | | |
|---|---|---|
| **(b)** **(ii)** | $$1 \cdot 96 \sqrt{\frac{\hat{p}(1-\hat{p})}{2500}} = 0 \cdot 01568$$ $$\Rightarrow \hat{p}(1-\hat{p}) = 2500 \left( \frac{0 \cdot 01568^2}{1 \cdot 96^2} \right)$$ $$\Rightarrow \hat{p}^2 - \hat{p} + \frac{4}{25} = 0$$ $$\hat{p} = \frac{1 \pm \sqrt{1 - 4\left(\frac{4}{25}\right)}}{2} = \frac{1 \pm \frac{3}{5}}{2}$$ $$\hat{p} = \frac{4}{5} \text{ or } \frac{1}{5}$$ $\frac{1}{5}$ outside the range $$\Rightarrow \hat{p} = \frac{4}{5}$$ | **Scale 10D(0, 3, 5, 8, 10)** *Low Partial Credit:* $\sqrt{\frac{\hat{p}(1-\hat{p})}{2500}}$ or equivalent written *Mid Partial Credit:* Formula fully substituted *High Partial Credit:* Quadratic in form $a\hat{p}^2 + b\hat{p} + c = 0$ |
| **(c)** | $H_0$: Mean weight of bags has not changed $H_1$: Mean weight of bags has changed $$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{13 \cdot 1 - 12}{\frac{4 \cdot 5}{\sqrt{80}}} = 2 \cdot 186$$ $$2 \cdot 186 > 1 \cdot 96$$ Mean weight of the bags has changed | **Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* CI formulated with some correct substitution 1·96 $H_0$ or $H_1$ *High Partial Credit:* z score fully substituted |

**(d)**

$$P(\text{weight} > 3000)$$

$$= P\left(\text{Average of those on bus} > \frac{3000}{40}\right)$$

$$P(\bar{x} > 75) = 1 - P(\bar{x} < 75)$$

$$z = \frac{75 - 73}{\frac{12}{\sqrt{40}}}$$

$$= 1 \cdot 054$$

This gives a proportion of $0 \cdot 8531$.

$1 - 0 \cdot 8531 = 0 \cdot 1469$

$$= 14 \cdot 69\%$$

This is the probability that the bus with 40 passengers will be above the maximum weight allowance.

**Scale 10D (0, 3, 5, 8, 10)**

*Low Partial Credit:*

$\frac{3000}{40}$

$\mu$ or $\sigma$ identified

*Mid Partial Credit:*

$z$ formula fully substituted

*High Partial Credit:*

$1 \cdot 054$

**(e)**

Median is $12 \cdot 5 \Longrightarrow D + E = 25$

LQ is $7 \cdot 5 \Longrightarrow B + C = 15$

IQR is 12 and $12 + 7 \cdot 5 = 19 \cdot 5$

$\Longrightarrow$ The upper quartile $= 19 \cdot 5$

$F + G = 39$

$G = 23$ so $F = 39 - 23 = 16$

Now $B + C + D + E + F + G = 79$

The total is $8 \times 13 \cdot 5 = 108$

So $A + H = 108 - 79 = 29$

$H - A = 21$ (range)

$A = 4$ and $H = 25$

$D + E = 25$ so $D = 11$, $E = 14$ (cannot be 12

and 13 also cannot be 10 and 15)

$B + C = 15$ so $B = 6$, $C = 9$ (cannot be 7 and

8 also cannot be 5 and 10)

The list is:

| A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| 4 | 6 | 9 | 11 | 14 | 16 | 23 | 25 |

**Scale 10D (0, 3, 5, 8, 10)**
*Low Partial Credit:*
One unknown number given
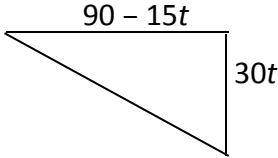One relevant equation written

*Mid Partial Credit*
Three unknown numbers given
Three relevant equations written

*High Partial Credit:*
Five unknown numbers given
Five relevant equations written

| Q9 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| **(a)** | $$d = \sqrt{\left(90 - \frac{15}{2}\right)^2 + \left(\frac{30}{2}\right)^2}$$ $$d = \sqrt{(82{\cdot}5)^2 + (15)^2}$$ $$d = 83{\cdot}85 \text{ km}$$ | **Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* $\frac{15}{2}$ or $\frac{30}{2}$ Indication of Pythagoras *High Partial Credit:* Pythagoras fully substituted |
| **(b)** |  $$s^2 = (90 - 15t)^2 + (30t)^2$$ $$s^2 = 8100 - 2700t + 225t^2 + 900t^2$$ $$s^2 = 1125t^2 - 2700t + 8100$$ $$s = (1125t^2 - 2700t + 8100)^{\frac{1}{2}}$$ | **Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* 90 – 15t  or 30t *High Partial Credit:* Pythagoras fully substituted |
| **(c)** | $$s = (1125t^2 - 2700t + 8100)^{\frac{1}{2}}$$ $$\frac{ds}{dt} = \frac{(2250t - 2700)}{2\sqrt{1125t^2 - 2700t + 8100}}$$ $$\Rightarrow 2250t - 2700 = 0$$ $$t = \frac{2700}{2250} = 1{\cdot}2 \text{ hours}$$ $$s = (1125t^2 - 2700t + 8100)^{\frac{1}{2}}$$ $$s = (1125(1{\cdot}2)^2 - 2700(1{\cdot}2) + 8100)^{\frac{1}{2}}$$ $$s = 80{\cdot}4984 \approx 80{\cdot}5 \text{ km}$$ | **Scale 5D(0, 2, 3, 4, 5)** *Low Partial Credit:* Any correct differntiation *Mid Partial Credit:* Value of t found *High Partial Credit:* Formula for *s* fully substituted Incorrect value of *t* (found through calculus) substituted and worked correctly. **Note**: No calculus $\Rightarrow$ 0 credit |

**Marcanna breise as ucht freagairt trí Ghaeilge**


**(Bonus marks for answering through Irish)**


Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar.   Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar.  Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc $\times$ 5% = 9·9 $\Rightarrow$ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná  225 marc, ríomhtar an bónas de réir na foirmle [300 – bunmharc] $\times$ 15%, agus an marc bónais sin a shlánú **síos**.  In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

| Bunmharc | Marc Bónais |
|----------|-------------|
| 226 | 11 |
| 227 – 233 | 10 |
| 234 – 240 | 9 |
| 241 – 246 | 8 |
| 247 – 253 | 7 |
| 254 – 260 | 6 |
| 261 – 266 | 5 |
| 267 – 273 | 4 |
| 274 – 280 | 3 |
| 281 – 286 | 2 |
| 287 – 293 | 1 |
| 294 – 300 | 0 |