



**Coimisiún na Scrúduithe Stáit**  
**State Examinations Commission**

**Leaving Certificate 2015**

**Marking Scheme**

**Mathematics**

**Higher Level**

### **Note to teachers and students on the use of published marking schemes**

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

### **Future Marking Schemes**

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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Coimisiún na Scrúduithe Stáit  
State Examinations Commission

Leaving Certificate Examination 2015

# Mathematics

Paper 1

Higher Level

Friday 5 June

Afternoon 2:00 – 4:30

300 marks

## Model Solutions – Paper 1

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

## Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

**You will lose marks if all necessary work is not clearly shown.**

**You may lose marks if the appropriate units of measurement are not included, where relevant.**

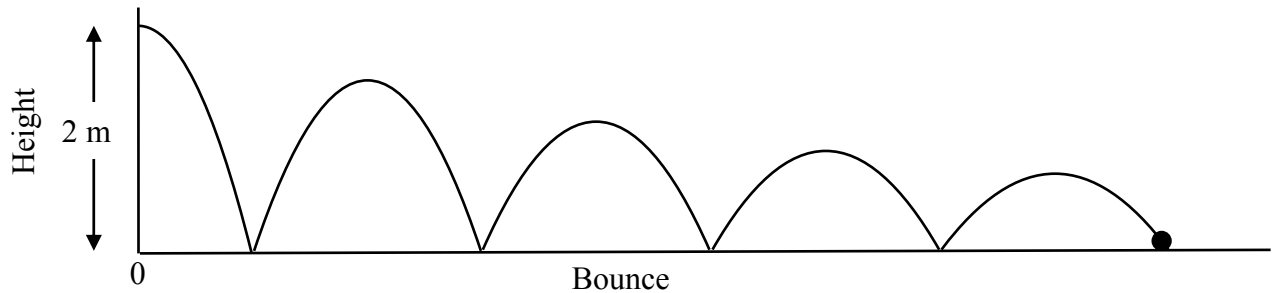
**You may lose marks if your answers are not given in simplest form, where relevant.**

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

**Question 1****(25 marks)**

Mary threw a ball onto level ground from a height of 2 m. Each time the ball hit the ground it bounced back up to  $\frac{3}{4}$  of the height of the previous bounce, as shown.



- (a) Complete the table below to show the maximum height, **in fraction form**, reached by the ball on each of the first four bounces.

Bounce	0	1	2	3	4
Height (m)	$\frac{2}{1}$	$\frac{3}{2}$	$\frac{9}{8}$	$\frac{27}{32}$	$\frac{81}{128}$

- (b) Find, in metres, the total vertical distance (up and down) the ball had travelled when it hit the ground for the 5<sup>th</sup> time. Give your answer in fraction form.

$$2 + 2\left(\frac{3}{2} + \frac{9}{8} + \frac{27}{32} + \frac{81}{128}\right) = 2 + 2\left(\frac{525}{128}\right) = \frac{653}{64} = 10\frac{13}{64} \text{ m}$$

or

$$\begin{aligned} 2 + 2\left(\frac{3}{2} + \frac{9}{8} + \frac{27}{32} + \frac{81}{128}\right) &= 2 + 2S_4 \\ &= 2 + 2\left(\frac{\frac{3}{2}(1 - (\frac{3}{4})^4)}{1 - \frac{3}{4}}\right) \\ &= 2 + \frac{525}{64} = \frac{653}{64} = 10\frac{13}{64} \text{ m} \end{aligned}$$

- (c) If the ball were to continue to bounce indefinitely, find, in metres, the total vertical distance it would travel.

$$\begin{aligned}2 + 2\left(\frac{3}{2} + \frac{9}{8} + \dots\right) &= 2 + 2\left(\frac{a}{1-r}\right) \\ &= 2 + 2\left(\frac{\frac{3}{2}}{1-\frac{3}{4}}\right) \\ &= 2 + 12 = 14 \text{ m}\end{aligned}$$



**Question 2****(25 marks)**Solve the equation  $x^3 - 3x^2 - 9x + 11 = 0$ .Write any irrational solution in the form  $a + b\sqrt{c}$ , where  $a, b, c \in \mathbb{Z}$ .

$$f(x) = x^3 - 3x^2 - 9x + 11$$

$$f(1) = 1^3 - 3(1)^2 - 9 + 11 = 0$$

$\Rightarrow x = 1$  is a solution.

$(x - 1)$  is a factor

$$\begin{array}{r} x-1 \overline{) \begin{array}{r} x^3 - 3x^2 - 9x + 11 \\ x^3 - x^2 \\ \hline -2x^2 - 9x + 11 \\ -2x^2 + 2x \\ \hline -11x + 11 \\ -11x + 11 \\ \hline 0 \end{array}} \end{array}$$

**or**

$$(x-1)(x^2 + Ax - 11) = x^3 - 3x^2 - 9x + 11$$

$$\Rightarrow x^3 + Ax^2 - x - x^2 - Ax + 11 = x^3 - 3x^2 - 9x + 11$$

$$\Rightarrow A - 1 = -3$$

$$\Rightarrow A = -2$$

**or**

	$x^2$	$-2x$	$-11$
$x$	$x^3$	$-2x^2$	$-11x$
$-1$	$-x^2$	$2x$	$11$

Hence, other factor is  $x^2 - 2x - 11$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)} = \frac{2 \pm \sqrt{48}}{2} = \frac{2 \pm 4\sqrt{3}}{2} = 1 \pm 2\sqrt{3}$$

Solutions:  $\{1, 1 + 2\sqrt{3}, 1 - 2\sqrt{3}\}$

**Question 3****(25 marks)**Let  $f(x) = -x^2 + 12x - 27$ ,  $x \in \mathbb{R}$ .**(a) (i)** Complete Table 1 below.

Table 1							
$x$	3	4	5	6	7	8	9
$f(x)$	0	5	8	9	8	5	0

**(ii)** Use Table 1 and the trapezoidal rule to find the approximate area of the region bounded by the graph of  $f$  and the  $x$ -axis.

$$\begin{aligned}
 A &= \frac{h}{2} [y_1 + y_n + 2(y_2 + y_3 + \dots + y_{n-1})] \\
 &= \frac{1}{2} [0 + 0 + 2(5 + 8 + 9 + 8 + 5)] \\
 &= 35 \text{ square units}
 \end{aligned}$$

**(b) (i)** Find  $\int_3^9 f(x) dx$ .

$$\begin{aligned}
 &\int_3^9 (-x^2 + 12x - 27) dx \\
 &= \left[ \frac{-x^3}{3} + \frac{12x^2}{2} - 27x \right]_3^9 \\
 &= (-243 + 486 - 243) - (-9 + 54 - 81) \\
 &= 36
 \end{aligned}$$

**(ii)** Use your answers above to find the percentage error in your approximation of the area, correct to one decimal place.

$$\frac{1}{36} \times 100 = 2.8\%$$

**Question 4****(25 marks)**

- (a) The complex numbers  $z_1$ ,  $z_2$  and  $z_3$  are such that  $\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3}$ ,  $z_2 = 2 + 3i$  and  $z_3 = 3 - 2i$ , where  $i^2 = -1$ . Write  $z_1$  in the form  $a + bi$ , where  $a, b \in \mathbb{Z}$ .

$$\begin{aligned} \frac{2}{z_1} &= \frac{1}{z_2} + \frac{1}{z_3} = \frac{1}{2+3i} + \frac{1}{3-2i} \\ &= \frac{3-2i+2+3i}{(2+3i)(3-2i)} = \frac{5+i}{12+5i} \\ \Rightarrow \frac{z_1}{2} &= \frac{12+5i}{5+i} \\ &= \frac{12+5i}{5+i} \times \frac{5-i}{5-i} \\ &= \frac{65+13i}{26} \\ \Rightarrow z_1 &= 5+i \end{aligned}$$

**or**

$$\begin{aligned} \frac{1}{2+3i} &= \frac{1}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{2-3i}{4+9} = \frac{2-3i}{13} \\ \frac{1}{3-2i} &= \frac{1}{3-2i} \cdot \frac{3+2i}{3+2i} = \frac{3+2i}{4+9} = \frac{3+2i}{13} \\ \frac{1}{2+3i} + \frac{1}{3-2i} &= \frac{2-3i}{13} + \frac{3+2i}{13} = \frac{5-i}{13} \\ \frac{2}{z_1} &= \frac{5-i}{13} \end{aligned}$$

Let  $z_1 = a + bi$

$$\frac{2}{a+bi} = \frac{5-i}{13}$$

$$26 = (5-i)(a+bi)$$

$$26 + (0)i = 5a + 5bi - ai + b$$

$$26 + (0)i = (5a+b) + (-a+5b)i$$

$$\Rightarrow 5a + b = 26 \dots(i) \text{ and } -a + 5b = 0 \dots(ii)$$

$$(i): \quad 5a + b = 26$$

$$(ii): \quad \frac{-5a + 25b = 0}{26b = 26}$$

$$b = 1$$

$$\text{From (ii): } 5b = a$$

$$\Rightarrow a = 5$$

$$z_1 = 5 + i$$

- (b) Let  $\omega$  be a complex number such that  $\omega^n = 1$ ,  $\omega \neq 1$ , and  $S = 1 + \omega + \omega^2 + \dots + \omega^{n-1}$ . Use the formula for the sum of a finite geometric series to write the value of  $S$  in its simplest form.

$$S = 1 + \omega + \omega^2 + \dots + \omega^{n-1}$$

$$a = 1, \quad r = \omega$$

$$S = \frac{1(1 - \omega^n)}{1 - \omega} = \frac{1(1 - 1)}{1 - \omega} = 0$$

**Question 5****(25 marks)**

- (a) Solve the equation  $x = \sqrt{x+6}$ ,  $x \in \mathbb{R}$ .

$$\begin{aligned}x &= \sqrt{x+6} \\ \Rightarrow x^2 &= x+6 \\ \Rightarrow x^2 - x - 6 &= 0 \\ \Rightarrow (x+2)(x-3) &= 0 \\ \Rightarrow x &= -2, \quad x = 3 \\ x = -2: \quad -2 &\neq \sqrt{-2+6} = \sqrt{4} = 2 \quad \times \\ x = 3: \quad 3 &= \sqrt{3+6} = \sqrt{9} = 3 \quad \checkmark\end{aligned}$$

- (b) Differentiate  $x - \sqrt{x+6}$  with respect to  $x$ .

$$\begin{aligned}f(x) &= x - \sqrt{x+6} = x - (x+6)^{\frac{1}{2}} \\ f'(x) &= 1 - \frac{1}{2}(x+6)^{-\frac{1}{2}} = 1 - \frac{1}{2\sqrt{x+6}}\end{aligned}$$

- (c) Find the co-ordinates of the turning point of the function  $y = x - \sqrt{x+6}$ ,  $x \geq -6$ .

$$\begin{aligned}f'(x) = 0 &\Rightarrow 1 - \frac{1}{2\sqrt{x+6}} = 0 \\ &\Rightarrow 2\sqrt{x+6} = 1 \\ &\Rightarrow x+6 = \frac{1}{4} \\ &\Rightarrow x = -5\frac{3}{4} \\ f(-5\frac{3}{4}) &= -5\frac{3}{4} - \sqrt{\frac{1}{4}} = -6\frac{1}{4} \\ &(-5\frac{3}{4}, -6\frac{1}{4})\end{aligned}$$

**Question 6****(25 marks)**

- (a) Donagh is arranging a loan and is examining two different repayment options.
- (i) Bank A will charge him a monthly interest rate of 0.35%. Find, correct to three significant figures, the annual percentage rate (APR) that is equivalent to a monthly interest rate of 0.35%.

$$F = P(1 + i)^t = 1(1 + 0.0035)^{12} = 1.042818$$
$$\Rightarrow i = 4.28\%$$

- (ii) Bank B will charge him a rate that is equivalent to an APR of 4.5%. Find, correct to three significant figures, the monthly interest rate that is equivalent to an APR of 4.5%.

$$F = P(1 + i)^t$$
$$1.045 = 1(1 + i)^{12}$$
$$1 + i = \sqrt[12]{1.045} = 1.0036748$$
$$\Rightarrow i = 0.367\%$$

- (b) Donagh borrowed €80 000 at a monthly interest rate of 0.35%, fixed for the term of the loan, from Bank A. The loan is to be repaid in equal monthly repayments over ten years. The first repayment is due one month after the loan is issued. Calculate, correct to the nearest euro, the amount of each monthly repayment.

$$\begin{aligned}
 A &= P \left[ \frac{i(1+i)^t}{(1+i)^t - 1} \right] \\
 &= 80000 \left[ \frac{0.0035(1.0035)^{120}}{(1.0035)^{120} - 1} \right] \\
 &= 80000 \left[ \frac{0.00532296}{0.520846} \right] \\
 &= 817.59 = \text{€}818
 \end{aligned}$$

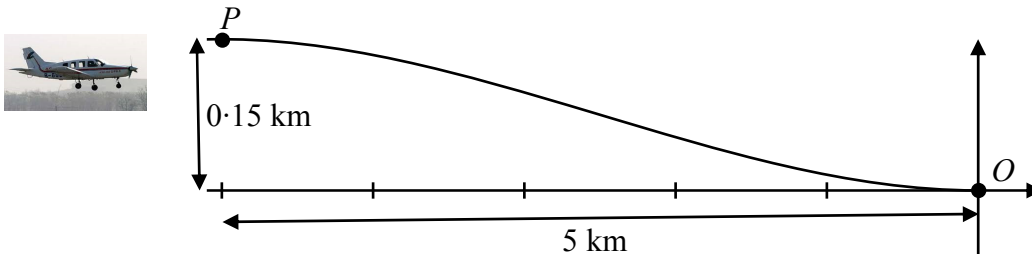
or

$$\begin{aligned}
 80000 &= \frac{A}{1.0035} + \frac{A}{1.0035^2} + \dots + \frac{A}{1.0035^{120}} \\
 &= A \left[ \frac{1}{1.0035} + \frac{1}{1.0035^2} + \dots + \frac{1}{1.0035^{120}} \right] \\
 &= A \left[ \frac{\frac{1}{1.0035} \left( 1 - \left( \frac{1}{1.0035} \right)^{120} \right)}{1 - \frac{1}{1.0035}} \right] \\
 &= A \left[ \frac{0.342471198}{0.0035} \right] \\
 &= A [ 97.8489137 ] \\
 A &= 817.58 = \text{€}818
 \end{aligned}$$

Answer **all three** questions from this section.

**Question 7****(50 marks)**

A plane is flying horizontally at  $P$  at a height of 150 m above level ground when it begins its descent.  $P$  is 5 km, horizontally, from the point of touchdown  $O$ . The plane lands horizontally at  $O$ .



Taking  $O$  as the origin,  $(x, f(x))$  approximately describes the path of the plane's descent where  $f(x) = 0.0024x^3 + 0.018x^2 + cx + d$ ,  $-5 \leq x \leq 0$ , and both  $x$  and  $f(x)$  are measured in km.

- (a) (i) Show that  $d = 0$ .

$$f(x) = 0.0024x^3 + 0.018x^2 + cx + d$$

$$f(0) = 0 + 0 + 0 + d = 0 \Rightarrow d = 0$$

- (ii) Using the fact that  $P$  is the point  $(-5, 0.15)$ , or otherwise, show that  $c = 0$ .

$$f(x) = 0.0024x^3 + 0.018x^2 + cx$$

$$f(-5) = 0.0024(-5)^3 + 0.018(-5)^2 + c(-5) = 0.15$$

$$\Rightarrow 0.15 - 5c = 0.15 \Rightarrow c = 0$$

or

$$\text{The plane lands horizontally at } O \Rightarrow f'(x) = 0 \text{ when } x = 0$$

$$f'(x) = 0.0072x^2 + 0.036x + c$$

$$f'(0) = 0 + 0 + c = 0$$

$$\Rightarrow c = 0$$

- (b) (i) Find the value of  $f'(x)$ , the derivative of  $f(x)$ , when  $x = -4$ .

$$f(x) = 0.0024x^3 + 0.018x^2 + cx + d$$

$$f'(x) = 0.0072x^2 + 0.036x$$

$$f'(-4) = 0.0072(-4)^2 + 0.036(-4)$$

$$= -0.0288$$



- (ii) Use your answer to part (b) (i) above to find the angle at which the plane is descending when it is 4 km from touchdown. Give your answer correct to the nearest degree.

$$\tan \theta = f'(x) = -0.0288 \Rightarrow \theta = 178.3503^\circ$$

$$\text{Angle of descent } \alpha = 1.6497^\circ = 2^\circ$$

- (c) Show that  $(-2.5, 0.075)$  is the point of inflection of the curve  $y = f(x)$ .

$$f'(x) = 0.0072x^2 + 0.036x$$

$$f''(x) = 0.0144x + 0.036 = 0$$

$$\Rightarrow x = -2.5$$

$$f(x) = 0.0024x^3 + 0.018x^2$$

$$f(-2.5) = 0.0024(-2.5)^3 + 0.018(-2.5)^2$$

$$= -0.0375 + 0.1125 = 0.075$$

$$(-2.5, 0.075)$$

- (d) (i) If  $(x, y)$  is a point on the curve  $y = f(x)$ , verify that  $(-x-5, -y+0.15)$  is also a point on  $y = f(x)$ .

$$f(x) = 0.0024x^3 + 0.018x^2$$

$$f(-x-5) = 0.0024(-x-5)^3 + 0.018(-x-5)^2$$

$$= 0.0024(-x^3 - 15x^2 - 75x - 125) + 0.018(x^2 + 10x + 25)$$

$$= -0.0024x^3 - 0.018x^2 + 0x + 0.15$$

$$= -y + 0.15$$

- (ii) Find the image of  $(-x-5, -y+0.15)$  under symmetry in the point of inflection.

Point:  $(-x-5, -y+0.15)$   
 Point of inflection:  $(-2.5, 0.075)$

Change in  $x$  value:  $(-2.5) - (-x-5) = x+2.5$   
 Change in  $y$  value:  $0.075 - (-y+0.15) = y-0.075$

Image of point of inflection:  
 $x$  value:  $-2.5 + (x+2.5) = x$   
 $y$  value:  $0.075 + (y-0.075) = y$   
 $\Rightarrow (x, y)$  is image

**or**

Let  $(x, y)$  be the image.

$$\left( \frac{-x-5+x}{2}, \frac{-y+0.15+y}{2} \right) = (-2.5, 0.075), \text{ the point of inflection}$$

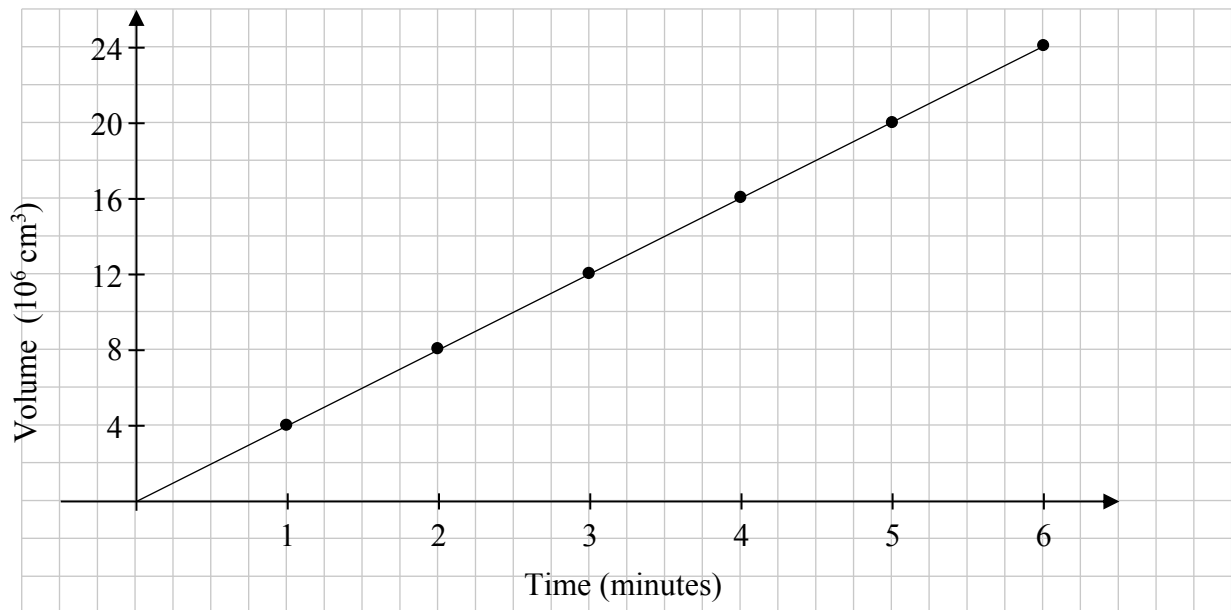
**Question 8****(50 marks)**

An oil-spill occurs off-shore in an area of calm water with no currents. The oil is spilling at a rate of  $4 \times 10^6 \text{ cm}^3$  per minute. The oil floats on top of the water.

- (a) (i) Complete the table below to show the total volume of oil on the water after each of the first 6 minutes of the oil-spill.

Time (minutes)	1	2	3	4	5	6
Volume ( $10^6 \text{ cm}^3$ )	4	8	12	16	20	24

- (ii) Draw a graph to show the total volume of oil on the water over the first 6 minutes.



- (iii) Write an equation for  $V(t)$ , the volume of oil on the water, in  $\text{cm}^3$ , after  $t$  minutes.

Line, slope  $4 \times 10^6$ , passing through  $(0, 0)$ .

$$V(t) = (4 \times 10^6) t$$

- (b) The spilled oil forms a circular oil slick **1 millimetre** thick.

- (i) Write an equation for the volume of oil in the slick, in  $\text{cm}^3$ , when the radius is  $r$  cm.

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi r^2 (0.1) \\ &= 0.1 \pi r^2 \text{ cm}^3 \end{aligned}$$

- (ii) Find the rate, in cm per minute, at which the radius of the oil slick is increasing when the radius is 50 m.

$$\begin{aligned}\frac{dV}{dt} &= 4 \times 10^6 \text{ cm}^3 \text{ per minute} \\ V &= \pi r^2 h \text{ where } h = 0.1 \text{ cm} \\ \frac{dV}{dr} &= 2\pi r h \\ \frac{dV}{dr} &= 0.2\pi r \\ \frac{dr}{dt} &= \frac{dr}{dV} \frac{dV}{dt} = \frac{1}{0.2\pi r} \times 4 \times 10^6 \\ &= \frac{4 \times 10^6}{0.2\pi(5000)} = 1273.3 \text{ cm per minute}\end{aligned}$$

- (c) Show that the area of water covered by the oil slick is increasing at a constant rate of  $4 \times 10^7 \text{ cm}^2$  per minute.

$$\begin{aligned}A &= \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r \\ \frac{dA}{dt} &= \frac{dA}{dr} \frac{dr}{dt} = 2\pi r \frac{4 \times 10^6}{0.2\pi r} = 4 \times 10^7 \text{ cm}^2 \text{ per minute}\end{aligned}$$

or

$$\begin{aligned}(0.1)\pi r^2 &= (4 \times 10^6)t \\ \Rightarrow A &= \pi r^2 = (4 \times 10^7)t \\ \frac{dA}{dt} &= 4 \times 10^7\end{aligned}$$

- (d) The nearest land is 1 km from the point at which the oil-spill began. Find how long it will take for the oil slick to reach land. Give your answer correct to the nearest hour.

$$\begin{aligned}A &= \pi r^2 = \pi(10^5)^2 = \pi 10^{10} \text{ cm}^2 \\ t &= \frac{\pi 10^{10}}{4 \times 10^7} = \frac{\pi 10^3}{4} = 785.398 \text{ minutes} \\ &= 13.09 = 13 \text{ hours}\end{aligned}$$

**Question 9****(50 marks)**

The approximate length of the day in Galway, measured in hours from sunrise to sunset, may be calculated using the function

$$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right),$$

where  $t$  is the number of days after March 21<sup>st</sup> and  $\left(\frac{2\pi}{365}t\right)$  is expressed in radians.

- (a) Find the length of the day in Galway on June 5<sup>th</sup> (76 days after March 21<sup>st</sup>). Give your answer in hours and minutes, correct to the nearest minute.

$$\begin{aligned} f(t) &= 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right) \\ f(76) &= 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365} \times 76\right) \\ &= 12 \cdot 25 + 4 \cdot 587 = 16 \cdot 837 = 16 \text{ hours } 50 \text{ minutes} \end{aligned}$$

- (b) Find a date on which the length of the day in Galway is approximately 15 hours.

$$\begin{aligned} f(t) &= 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right) = 15 \\ \Rightarrow \sin\left(\frac{2\pi}{365}t\right) &= 0 \cdot 578947 \\ \Rightarrow \frac{2\pi}{365}t &= 0 \cdot 6174371 \\ \Rightarrow t &= 35 \cdot 87 \\ &36 \text{ days after March 21 is April 26.} \end{aligned}$$

- (c) Find  $f'(t)$ , the derivative of  $f(t)$ .

$$\begin{aligned} f(t) &= 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right) \\ f'(t) &= 0 + 4 \cdot 75 \times \frac{2\pi}{365} \cos\left(\frac{2\pi}{365}t\right) \\ &= \frac{9 \cdot 5\pi}{365} \cos\left(\frac{2\pi}{365}t\right) \end{aligned}$$

- (d) Hence, or otherwise, find the length of the longest day in Galway.

$$f(t) \text{ is a maximum when } \sin\left(\frac{2\pi}{365}t\right) \text{ is a maximum of 1.}$$
$$t = 12 \cdot 25 + 4 \cdot 75 = 17 \text{ hours}$$

or

$$f'(t) = 0 \Rightarrow \frac{9 \cdot 5\pi}{365} \cos\left(\frac{2\pi}{365}t\right) = 0$$
$$\Rightarrow \cos\left(\frac{2\pi}{365}t\right) = 0$$
$$\Rightarrow \frac{2\pi}{365}t = \frac{\pi}{2}$$
$$\Rightarrow t = \frac{365}{4} = 91 \cdot 25$$
$$f(91 \cdot 25) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365} \times 91 \cdot 25\right)$$
$$= 12 \cdot 25 + 4 \cdot 75 \sin \frac{\pi}{2}$$
$$= 17 \text{ hours}$$

- (e) Use integration to find the average length of the day in Galway over the six months from March 21<sup>st</sup> to September 21<sup>st</sup> (184 days). Give your answer in hours and minutes, correct to the nearest minute.

$$\frac{1}{b-a} \int_a^b f(x) dx = \frac{1}{184} \int_0^{184} \left(12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right)\right) dt$$
$$= \frac{1}{184} \left[12 \cdot 25t - 4 \cdot 75 \times \frac{365}{2\pi} \cos\left(\frac{2\pi}{365}t\right)\right]_0^{184}$$
$$= \frac{1}{184} [(2254 + 275 \cdot 843) - (0 - 275 \cdot 934)]$$
$$= \frac{1}{184} [2805 \cdot 777]$$
$$= 15 \cdot 24879$$
$$= 15 \text{ hours } 15 \text{ minutes}$$

## Marking Scheme – Paper 1, Section A and Section B

### Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 4, 5		
10 mark scales	0, 10	0, 5, 10	0, 4, 8, 10	0, 2, 5, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

### Marking scales – level descriptors

#### A-scales (two categories)

- incorrect response
- correct response

#### B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

#### C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

#### D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

#### E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale 10C*, 9 marks may be awarded.

Throughout the scheme indicate by use of \* where an arithmetic error occurs.

## Summary of mark allocations and scales to be applied

### *Section A*

Question 1	
(a)	5C
(b)	10C
(c)	10C
Question 2	
	25E
Question 3	
(a)	15D
(b)	10C
Question 4	
(a)	15D
(b)	10C
Question 5	
(a)	10C
(b)	5B
(c)	10C
Question 6	
(a)(i)+(ii)	10C
(b)(	15C

### *Section B*

Question 7	
(a)(i)	5B
(a)(ii)	5B
(b)(i)	10C
(b)(ii)	5B
(c)	10D
(d)(i)	5C
(d)(ii)	10C
Question 8	
(a)(i)	5B
(a)(ii)	5B
(a)(iii)	5B
(b)(i)	5B
(b)(ii)	10D
(c)	10C
(d)	10C
Question 9	
(a)	10C
(b)	10C
(c)	10B
(d)	10D
(e)	10



## Detailed marking notes

**NOTE:** In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded.

Rounding and units penalty to be applied only once in each section (a), (b), (c) etc.

Throughout the scheme indicate by use of \* where an arithmetic error occurs.

### Section A

#### Question 1

(a) Scale 5C (0, 2, 4, 5)

*Low Partial Credit:*

- Any term correct

*High Partial Credit:*

- Any two terms correct

**Note:** Dividing by  $\frac{3}{4}$  gets high partial credit at most.

Correct decimal values high partial at most.

(b) Scale 10C (0, 4, 8, 10) – **NOTE:** two solutions

1<sup>st</sup> solution

*Low Partial Credit:*

- Indicates addition of terms

*High Partial Credit:*

- Recognises double distance after first hop
- Sum of all rises or drops

**or**

2<sup>nd</sup> solution

*Low Partial Credit:*

- Indicates addition of terms
- Indicates Geometric Progression

*High Partial Credit:*

- Correct Geometric Progression formula with correct substitution

(c) Scale 10C (0, 4, 8, 10)

*Low Partial Credit:*

- Recognition of sum to infinity
- $S_{\infty}$  formula

*High Partial Credit*

- Correct formula with correct substitution
- Sum of all rises or drops

## Question 2

(a) Scale 25E (0, 5, 10, 15, 20, 25)

*Low Partial Credit:*

- Effort at finding root, i.e.  $f(1)$ ,  $f(-1)$ , etc.

*Low Mid Partial Credit:*

- Finds one root correctly
- $x^2$  after division by incorrect factor
- Correct answers in decimal form from calculator with or without work

*High Mid Partial Credit:*

- Tries division and gets  $x^2$  at very minimum

*High Partial Credit:*

- Having got a quadratic equation with no remainder, fills in quadratic formula
- $1 \pm \sqrt{12}$

**Note:** If there is a remainder after division can only get maximum of 15 marks.

### Question 3

#### (a)(i) and (ii) combined

Scale 15D (0, 4, 7, 11, 15)

*Low Partial Credit:*

- Any one correct value
- Writes formula

*Mid Partial Credit:*

- Correct table

*High Partial Credit:*

- Correct formula for trapezoidal rule, and some correct substitution with  $h = 1$
- Completely incorrect table but applied correctly in a(ii)
- Correct table and 35 without work

**Note (1):** Answers in terms of  $h$  merit Mid Partial at most.

**Note (2):** Correct formula and some substitution gets High Partial.

**Note (3):** No formula and  $\frac{1}{2}[5 + 5 + 2(8 + 9 + 8)] = 30$  gets High Partial.

#### (b)(i) and (ii) combined

Scale 10C (0, 4, 8, 10)

*Low Partial Credit:*

- Any correct integration
- Correct substitution of  $f(x)$
- Correct % error formula
- Correct substitution of  $f(x)$  i.e.  $(-x^2 + 12x - 27)$

*High Partial Credit:*

- Correct integration with some correct substitution
- 97.2%

*Full Credit:*

- Accept 2.8% without work for full credit.

#### Question 4

(a) Scale 15D (0, 4, 7, 11, 15) – **NOTE:** two solutions

*Low Partial Credit:*

- Some rationalisation
- Some relevant rearrangement.

*Mid Partial Credit:*

- Gets  $z_1$  or  $\frac{2}{z_1}$  in the form of  $\frac{a+bi}{c+di}$

*High Partial Credit:*

- Correct use of conjugate in  $\frac{12+5i}{5+i}$

**or**

*Low Partial Credit:*

- One complex number correct

*Mid Partial Credit:*

- Two complex numbers correct

*High Partial Credit:*

- Correct use of conjugate in  $\frac{2}{a+bi} = \frac{5-i}{13}$

(b) Scale 10C (0, 4, 8, 10)

*Low Partial Credit:*

- Correct Geometric Progression formula
- Correct first term
- Correct ratio

*High Partial Credit:*

- Values substituted in formula

### Question 5

- (a) Scale 10C (0, 4, 8, 10)  
*Low Partial Credit:*
- Indication of squaring

*High Partial Credit:*

- Correct roots

**Note:** must indicate required root

- (b) Scale 5B (0, 2, 5)  
*Partial Credit:*
- Any correct differentiation
  - Indication of  $(x + 6)^{\frac{1}{2}}$

- (c) Scale 10C (0, 4, 8, 10)  
*Low Partial Credit:*
- Differentiation equals 0
- High Partial Credit:*
- Finds  $x$  value.

**Note (1):** A linear equation from  $f'(x)$  gets low partial at most.

**Note (2):** Must put  $f'(x) = 0$  in (c) to get any marks.

**Note (3):**  $f'(x)$  only and  $f''(x)$  only  $\Rightarrow$  .no credit

## Question 6

### (a)(i) and (ii) combined

Scale 10C (0, 4, 8, 10)

*Low Partial Credit:*

- Correct formula in either part
- Correct substitution in incorrect formula

*High Partial Credit:*

- Any one section correct

**Note:** Rate as 0.367% or 0.00367 gets High Partial.

### (b) Scale 15C (0, 5, 10, 15) – **NOTE:** two solutions

1<sup>st</sup> solution

*Low Partial Credit:*

- Any correct step, i.e. correct formula

*High Partial Credit:*

- Substitution in correct formula.

**or**

2<sup>nd</sup> solution

*Low Partial Credit:*

- Correct equation.
- Listing some terms
- Some substitution

*High Partial Credit:*

- Complete substitution and effort at evaluation.

**Note:** If A and 80 000 interchanged and remainder of work correct, may get High Partial credit.

## Section B

### Question 7

(a)(i) Scale 5B (0, 2, 5)

*Partial Credit:*

- Recognises  $x = 0$

(a)(ii) Scale 5B (0, 2, 5) – **NOTE:** two solutions

1<sup>st</sup> solution

*Partial Credit:*

- Uses  $x = -5$  or  $f(x) = 0 \cdot 15$

*Full credit:*

- Begins with  $c = 0$  and shows  $f(-5) = 0 \cdot 15$  or similar

**or**

2<sup>nd</sup> solution

*Partial Credit:*

- Uses  $x = -5$
- Gets  $f'(x)$
- Uses  $f'(x) = 0$  when  $x = 0$

(b)(i) Scale 10C (0, 3, 7, 10)

*Low Partial Credit:*

- Any term correctly differentiated.

*High Partial Credit:*

- Correct differentiation

*Full credit:*

- $-\frac{18}{625}$  is a correct answer

(b)(ii) Scale 5B (0, 2, 5)

*Partial Credit:*

- Recognition of connection between slope and  $\tan \theta$
- Any right angled triangle

(c) Scale 10D (0, 2, 5, 8, 10)

*Low Partial Credit:*

- Some correct differentiation of  $f'(x)$
- Mention of  $f'(x)$

*Mid Partial Credit:*

- Correct  $f''(x) = 0$

*High Partial Credit:*

- Value of  $x$  substituted

**(d)(i)** Scale 5C (0, 2, 4, 5)

*Low Partial Credit:*

- Some correct substitution

*High Partial Credit:*

- Correct expansions

**(d)(ii)** Scale 10C (0, 4, 8, 10) – **NOTE:** two solutions

1<sup>st</sup> solution

*Low Partial Credit:*

- Work leading to change in  $x$ -value or  $y$ -value

*High Partial Credit:*

- Correct change in  $x$  and  $y$  values

**or**

2<sup>nd</sup> solution

*Low Partial Credit:*

- Uses  $(x, y)$  as image, and no more

*High Partial Credit:*

- Effort at calculating mid-point

## Question 8

**(a)(i)** Scale 5B (0, 2, 5)

*Partial Credit:*

- One correct box

**(a)(ii)** Scale 5B (0, 2, 5)

*Partial Credit:*

- At least two points plotted

*No credit*

- Bar chart

**(a)(iii)** Scale 5B (0, 2, 5)

*Partial Credit:*

- Incomplete equation for volume
- $V =$  any function of  $t$
- Attempt at finding slope

**(b)(i)** Scale 5B (0, 2, 5)

*Partial Credit:*

- Correct volume formula
- Converting mm to cm



**(b)(ii)** Scale 10D (0, 2, 5, 8, 10)

*Low Partial Credit:*

- Mentions a relevant rate of change.

*Mid Partial Credit:*

- Gets  $\frac{dr}{dt}$  from  $\frac{dV}{dr}$  and  $\frac{dV}{dt}$
- Writing down chain rule.

*High Partial Credit:*

- Substitution of values

**(c)** Scale 10C (0, 4, 8, 10) – **NOTE:** two solutions

1<sup>st</sup> solution

*Low Partial Credit:*

- Mentions relevant rate of change.

*High Partial Credit:*

- States chain rule i.e.  $\frac{dA}{dt} = \frac{dA}{dr} \frac{dr}{dt}$

**or**

2<sup>nd</sup> solution

*Low Partial Credit:*

- Effort to establish value of  $A$

*High Partial Credit:*

- $A$  in terms of  $t$

**Note:** Must use calculus to get any credit.

**(d)** Scale 10C (0, 4, 8, 10)

*Low Partial Credit:*

- $r$  in centimetres
- Effort at expression of area

*High Partial Credit:*

- Correct expression for time

### Question 9

(a) Scale 10C (0, 4, 8, 10)

*Low Partial Credit:*

- Uses  $t = 76$

*High Partial Credit:*

- Correct substitution

**Note:** Using  $\pi = 90^\circ \Rightarrow$  one error, but do not penalise again in (b)

(b) Scale 10C (0, 4, 8, 10)

*Low Partial Credit:*

- Correct  $f(t)$
- $f(15)$  substituted.

*High Partial Credit:*

- Correct equation with  $t$  only

**Note:** Accept 35 or 36 substituted correctly and tested.

(c) Scale 10B (0, 5, 10)

*Partial Credit:*

- Any correct differentiation (note: '0' could be correct differentiation here)

**Note:** Substituting  $180^\circ$  for  $\pi \Rightarrow$  one error

(d) Scale 10D (0, 2, 5, 8, 10) – both solutions

*Low Partial Credit:*

- $f'(t) = 0$

*Mid Partial Credit:*

- Value of  $t$

*High Partial Credit:*

- Value of  $t$  substituted into  $f(t)$
- $f(t)$  maximum when  $\sin \theta = 1$

**Note:** Accept 91 or 92 substituted and evaluated correctly for full marks.

(e) Scale 10D (0, 2, 5, 8, 10)

*Low Partial Credit:*

- Correct expression in  $x$  or  $t$
- Correct formula
- Correct limits

*Mid Partial Credit:*

- Any correct integration

*High Partial Credit:*

- Correct integration and effort at substitution

**Note:** Integration with one error but finished correctly gets High Partial Credit.



Coimisiún na Scrúduithe Stáit  
State Examinations Commission

Leaving Certificate Examination 2015

# Mathematics

Paper 2

Higher Level

Monday 8 June      Morning 9:30 – 12:00

300 marks

## Model Solutions – Paper 2

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

## Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

**You will lose marks if all necessary work is not clearly shown.**

**You may lose marks if the appropriate units of measurement are not included, where relevant.**

**You may lose marks if your answers are not given in simplest form, where relevant.**

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

**Question 1****(25 marks)**

An experiment consists of throwing two fair, standard, six-sided dice and noting the sum of the two numbers thrown. If the sum is 9 or greater it is recorded as a “win” (W). If the sum is 8 or less it is recorded as a “loss” (L).

(a) Complete the table below to show all possible outcomes of the experiment.

		Die 2					
		1	2	3	4	5	6
Die 1	1	L	L	L	L	L	L
	2	L	L	L	L	L	L
	3	L	L	L	L	L	W
	4	L	L	L	L	W	W
	5	L	L	L	W	W	W
	6	L	L	W	W	W	W

(b) (i) Find the probability of a win on one throw of the two dice.

$$P(W) = \frac{10}{36} = \frac{5}{18}$$

(ii) Find the probability that each of 3 successive throws of the two dice results in a loss. Give your answer correct to four decimal places.

$$P(L, L, L) = \left(\frac{13}{18}\right)^3 = 0.3767$$

(c) The experiment is repeated until a total of 3 wins occur. Find the probability that the third win occurs on the tenth throw of the two dice. Give your answer correct to four decimal places.

$$P(2 \text{ wins in } 9) = \binom{9}{2} \left(\frac{5}{18}\right)^2 \left(\frac{13}{18}\right)^7$$

$$P(3 \text{ wins, 3rd on 10th throw}) = \binom{9}{2} \left(\frac{5}{18}\right)^2 \left(\frac{13}{18}\right)^7 \left(\frac{5}{18}\right) = 0.0791$$

**Question 2****(25 marks)**

A survey of 100 shoppers, randomly selected from a large number of Saturday supermarket shoppers, showed that the mean shopping spend was €90.45. The standard deviation of this sample was €20.73.

- (a) Find a 95% confidence interval for the mean amount spent in a supermarket on that Saturday.

$$\frac{\sigma}{\sqrt{n}} = \frac{20.73}{\sqrt{100}} = 2.073$$

$$\text{C. I.} = \bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}} = 90.45 \pm 4.06$$

We can be 95% confident that the mean amount spent was in the range  
 $\text{€}86.39 < \mu < \text{€}94.51$

- (b) A supermarket has claimed that the mean amount spent by shoppers on a Saturday is €94. Based on the survey, test the supermarket's claim using a 5% level of significance. Clearly state your null hypothesis, your alternative hypothesis, and your conclusion.

$H_0$  : Mean spend is €94

$H_1$  : Mean spend is not €94

**METHOD 1:**

$\bar{x} = 90.45$ ,  $\sigma = 20.73$ ,  $\mu = 94$ ,  $n = 100$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{90.45 - 94}{2.073} = -1.71$$

$$-1.71 > -1.96$$

Fail to reject null hypothesis ( Not enough evidence to reject the null hypothesis)

**or**

**METHOD 2:**

€94 is inside the confidence interval for the mean spend in the population

$\text{€}86.39 < \mu < \text{€}94.51$  worked out in part (i) etc.

Fail to reject null hypothesis ( Not enough evidence to reject the null hypothesis)

**Or**

**METHOD 3:**

C.I. based on a sample of 100 based on the claim is:

$$89.94 < \bar{x} < 98.06$$

€90.45 is inside this interval.

Fail to reject null hypothesis ( Not enough evidence to reject the null hypothesis)

- (c) Find the  $p$ -value of the test you performed in part (b) above and explain what this value represents in the context of the question.

$$\begin{aligned}P(z < -1.71) &= 1 - P(z < 1.71) \\ &= 1 - 0.9564 \\ &= 0.0436\end{aligned}$$

$$p\text{-value: } = 0.0436 \times 2 = 0.0872$$

Meaning: If the mean amount spent really was €94, then the probability that the sample mean would be €90.45 by chance is 8.72%. It is because this is more than a 5% chance that we do not reject the null hypothesis.

**Question 3****(25 marks)**

- (a)**
- The co-ordinates of two points are
- $A(4, -1)$
- and
- $B(7, t)$
- .

The line  $l_1 : 3x - 4y - 12 = 0$  is perpendicular to  $AB$ . Find the value of  $t$ .

$$\text{Slope } AB = \frac{t+1}{7-4} = \frac{t+1}{3} \qquad \text{Slope } l_1 = \frac{3}{4}$$

$$AB \perp l_1 \Rightarrow \frac{t+1}{3} \times \frac{3}{4} = -1 \Rightarrow t+1 = -4 \Rightarrow t = -5$$

**or**

$$AB : 4x + 3y + c = 0$$

$$(4, -1) \in 4x + 3y + c = 0 \Rightarrow 16 - 3 + c = 0 \Rightarrow c = -13$$

$$\therefore 4(7) + 3(t) - 13 = 0 \Rightarrow t = -5$$

- (b)**
- Find, in terms of
- $k$
- , the distance between the point
- $P(10, k)$
- and
- $l_1$
- .

$$d = \frac{|3(10) - 4k - 12|}{\sqrt{3^2 + 4^2}} = \frac{|18 - 4k|}{5}$$

- (c)**
- $P(10, k)$
- is on a bisector of the angles between the lines
- $l_1$
- and
- $l_2 : 5x + 12y - 20 = 0$
- .

- (i)**
- Find the possible values of
- $k$
- .

$$\begin{aligned} \left| \frac{18 - 4k}{5} \right| &= \left| \frac{50 + 12k - 20}{\sqrt{5^2 + 12^2}} \right| \\ \Rightarrow \left| \frac{18 - 4k}{5} \right| &= \left| \frac{30 + 12k}{13} \right| \\ \Rightarrow 13(18 - 4k) &= \pm 5(30 + 12k) \\ \Rightarrow -112k &= -84 \quad \text{or} \quad 8k = -384 \\ \Rightarrow k &= \frac{3}{4} \quad \text{or} \quad k = -48 \end{aligned}$$

- (ii)**
- If
- $k > 0$
- , find the distance from
- $P$
- to
- $l_1$
- .

$$k = \frac{3}{4} \Rightarrow d = \left| \frac{18 - 4\left(\frac{3}{4}\right)}{5} \right| = 3$$



**Question 4**

**(25 marks)**

Two circles  $s$  and  $c$  touch internally at  $B$ , as shown.

(a) The equation of the circle  $s$  is

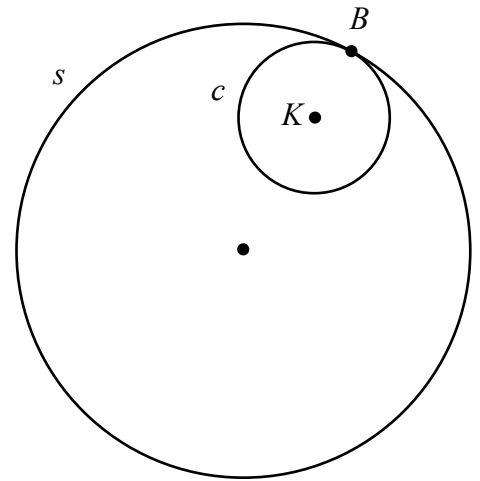
$$(x-1)^2 + (y+6)^2 = 360.$$

Write down the co-ordinates of the centre of  $s$ .

Centre: $(1, -6)$
-------------------

Write down the radius of  $s$  in the form  $a\sqrt{10}$ , where  $a \in \mathbb{N}$ .

Radius: $\sqrt{360} = 6\sqrt{10}$
-----------------------------------



- (b) (i) The point  $K$  is the centre of circle  $c$ .  
 The radius of  $c$  is one-third the radius of  $s$ .  
 The co-ordinates of  $B$  are  $(7, 12)$ .  
 Find the co-ordinates of  $K$ .

$ AK  :  KB  = 2 : 1$ $K\left(\frac{2 \times 7 + 1 \times 1}{2 + 1}, \frac{2 \times 12 + 1 \times -6}{2 + 1}\right) = (5, 6)$	<p><b>or</b></p> <p>Centre of <math>s</math> to <math>B</math> (translation)                  X ordinate goes up by 6                  Y ordinate goes up by 18  <math>\frac{2}{3}(6) + 1 = 5</math>  <math>\frac{2}{3}(18) - 6 = 6</math></p>
--	--

(ii) Find the equation of  $c$ .

$(x-5)^2 + (y-6)^2 = (2\sqrt{10})^2 = 40$
---

- (c) Find the equation of the common tangent at  $B$ .  
 Give your answer in the form  $ax + by + c = 0$ , where  $a, b, c \in \mathbb{Z}$ .

$\text{Slope } AB = \frac{12+6}{7-1} = \frac{18}{6} = 3$ $\text{Slope of tangent} = -\frac{1}{3}$ $\text{Equation: } y-12 = -\frac{1}{3}(x-7) \Rightarrow x+3y-43=0$
--

**Question 5**

**(25 marks)**

- (a) Prove that  $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ .

$$\begin{aligned}\tan(A + B) &= \frac{\sin(A + B)}{\cos(A + B)} \\ &= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \\ &= \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}} \\ &= \frac{\tan A + \tan B}{1 - \tan A \tan B}\end{aligned}$$

**or**

$$\begin{aligned}\frac{\tan A + \tan B}{1 - \tan A \tan B} &= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1 - \frac{\sin A \sin B}{\cos A \cos B}} \\ &= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B - \sin A \sin B}{\cos A \cos B}} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \\ &= \frac{\sin(A + B)}{\cos(A + B)} = \tan(A + B)\end{aligned}$$

- (b) Find all the values of  $x$  for which  $\sin(3x) = \frac{\sqrt{3}}{2}$ ,  $0 \leq x \leq 360$ ,  $x$  in degrees.

$$\begin{aligned}\sin 3x &= \frac{\sqrt{3}}{2} \\ \Rightarrow 3x &= 60^\circ, 120^\circ, 420^\circ, 480^\circ, 780^\circ, 840^\circ \\ \Rightarrow x &= 20^\circ, 40^\circ, 140^\circ, 160^\circ, 260^\circ, 280^\circ\end{aligned}$$

**or**

$$\begin{aligned}3x &= 60^\circ + n(360^\circ), n \in \mathbb{Z} \text{ or } 3x = 120^\circ + n(360^\circ), n \in \mathbb{Z} \\ x &= 20^\circ + n(120^\circ), n \in \mathbb{Z} \text{ or } x = 40^\circ + n(120^\circ), n \in \mathbb{Z}\end{aligned}$$

$$n=0 \Rightarrow x = 20^\circ \text{ or } x = 40^\circ$$

$$n=1 \Rightarrow x = 140^\circ \text{ or } x = 160^\circ$$

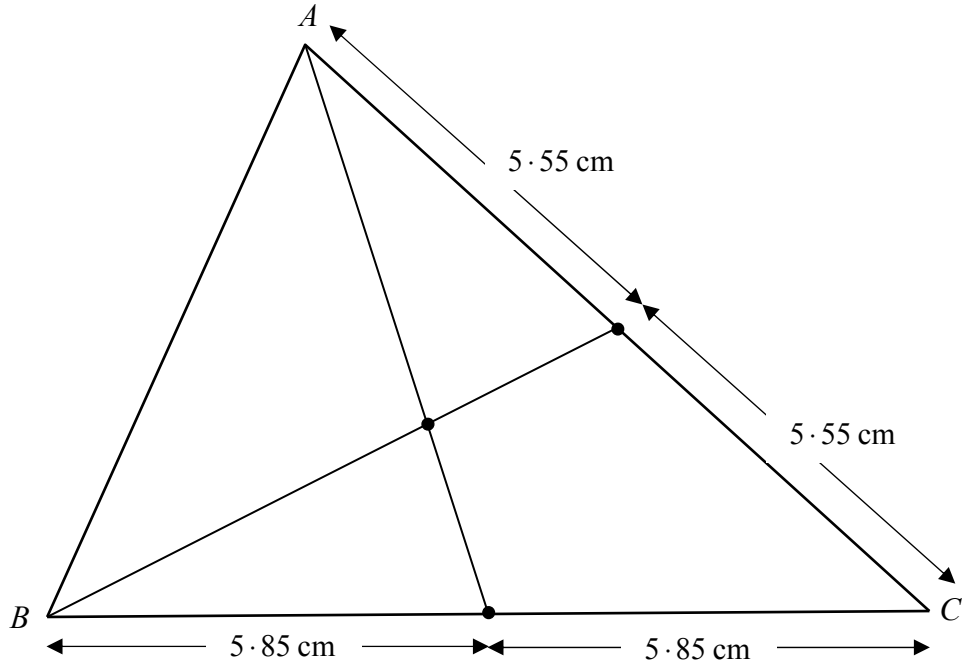
$$n=2 \Rightarrow x = 260^\circ \text{ or } x = 280^\circ$$

**Question 6**

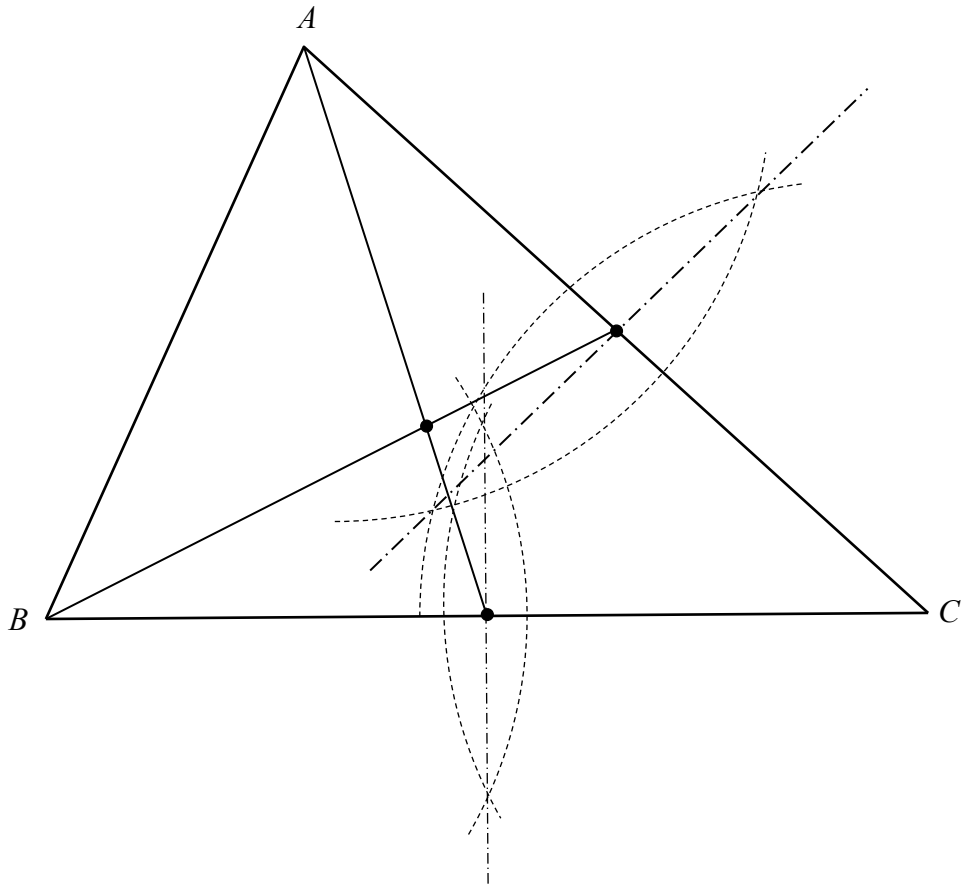
**(25 marks)**

- (a) Construct the centroid of the triangle  $ABC$  below. Show all construction lines.  
(Where measurement is used, show all relevant measurements and calculations clearly.)

$|AC| = 11.1 \text{ cm}$ ;  $|BC| = 11.7 \text{ cm}$

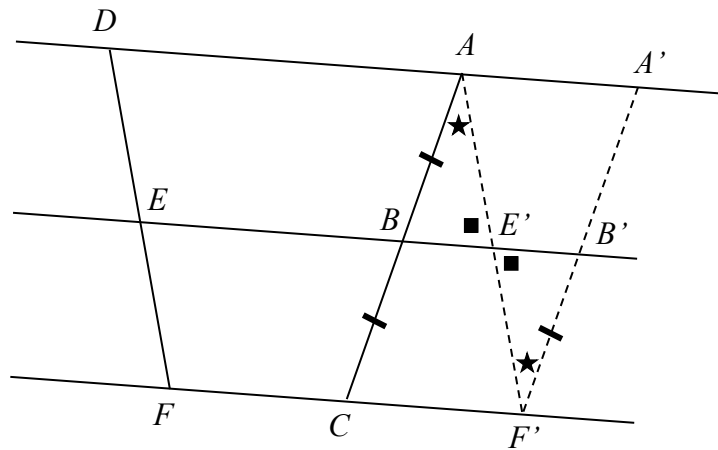


or



- (b) Prove that, if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal line.

Diagram:



Given:  $AD \parallel BE \parallel CF$ , as in the diagram, with  $|AB| = |BC|$

To Prove:  $|DE| = |EF|$

Construction: Draw  $AE' \parallel DE$ , cutting  $EB$  at  $E'$  and  $CF$  at  $F'$   
 Draw  $F'B' \parallel AB$ , cutting  $EB$  at  $B'$ , as in diagram.

Proof:

$ B'F'  =  BC $	(opposite sides in a parallelogram)
$=  AB $	(by assumption)
$ \angle BAE'  =  \angle E'F'B' $	(alternate angles)
$ \angle AE'B  =  \angle F'E'B' $	(vertically opposite angles)
$\therefore \triangle ABE'$ is congruent to $\triangle F'B'E'$	(ASA)
$\therefore  AE'  =  F'E' $	
But $ AE'  =  DE $ and $ F'E'  =  FE $	(opposite sides in a parallelogram)
$\therefore  DE  =  EF $	

Answer **all three** questions from this section.

**Question 7****(40 marks)**

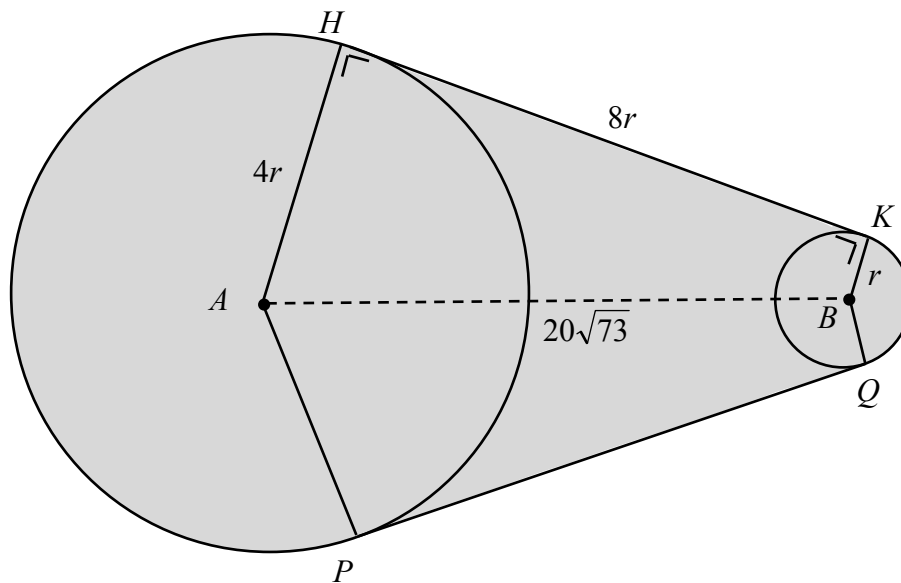
A flat machine part consists of two circular ends attached to a plate, as shown (diagram not to scale).

The sides of the plate,  $HK$  and  $PQ$ , are tangential to each circle.

The larger circle has centre  $A$  and radius  $4r$  cm.

The smaller circle has centre  $B$  and radius  $r$  cm.

The length of  $[HK]$  is  $8r$  cm and  $|AB| = 20\sqrt{73}$  cm.



- (a) Find  $r$ , the radius of the smaller circle. (Hint: Draw  $BT \parallel KH$ ,  $T \in AH$ .)

The diagram for part (a) is identical to the one above, but with an additional construction. A line segment  $BT$  is drawn from center  $B$  parallel to  $HK$ , intersecting the radius  $AH$  at point  $T$ . This construction creates a right-angled triangle  $ATB$  with the right angle at  $T$ . The length of  $AT$  is labeled as  $3r$ , and the length of  $BT$  is labeled as  $8r$ . The distance  $AB$  remains  $20\sqrt{73}$ .

$$|AT|^2 + |BT|^2 = |AB|^2 \Rightarrow (3r)^2 + (8r)^2 = (20\sqrt{73})^2$$

$$\Rightarrow 9r^2 + 64r^2 = 29200$$

$$\Rightarrow r^2 = 400 \Rightarrow r = 20 \text{ cm}$$

- (b) Find the area of the quadrilateral  $ABKH$ .

$$\begin{aligned} |ABKH| &= |BKHT| + |\Delta ABT| \\ &= 20 \times 160 + \frac{1}{2}(60)(160) \\ &= 8000 \text{ cm}^2 \end{aligned}$$

- (c) (i) Find  $|\angle HAP|$ , in degrees, correct to one decimal place.

$$\begin{aligned} \tan |\angle HAB| &= \frac{160}{60} \Rightarrow |\angle HAB| = 69.44^\circ \\ &\Rightarrow |\angle HAP| = 138.9^\circ \end{aligned}$$

- (ii) Find the area of the machine part, correct to the nearest  $\text{cm}^2$ .

$$\begin{aligned} &\text{Area large sector } HAP + 2 \text{ area } HABK + \text{area sector } KBQ \\ &= \pi(80)^2 \left( \frac{221.1}{360} \right) + 2 \times 8000 + \pi(20)^2 \left( \frac{138.9}{360} \right) \\ &= 12348.55 + 16000 + 484.85 \\ &= 28833.4 \\ &= 28833 \end{aligned}$$

**Question 8****(65 marks)**

In basketball, players often have to take free throws. When Michael takes his first free throw in any game, the probability that he is successful is 0.7.

For all subsequent free throws in the game, the probability that he is successful is:

- 0.8 if he has been successful on the previous throw
- 0.6 if he has been unsuccessful on the previous throw.

- (a) Find the probability that Michael is successful (S) with all three of his first three free throws in a game.

$$P(S, S, S) = 0.7 \times 0.8 \times 0.8 = 0.448$$

- (b) Find the probability that Michael is unsuccessful (U) with his first two free throws and successful with the third.

$$P(U, U, S) = 0.3 \times 0.4 \times 0.6 = 0.072$$

- (c) List all the ways that Michael could be successful with his third free throw in a game and hence find the probability that Michael is successful with his third free throw.

S, S, S    U, U, S    S, U, S    U, S, S

$$P(S, S, S) = 0.7 \times 0.8 \times 0.8 = 0.448$$

$$P(U, U, S) = 0.3 \times 0.4 \times 0.6 = 0.072$$

$$P(S, U, S) = 0.7 \times 0.2 \times 0.6 = 0.084$$

$$P(U, S, S) = 0.3 \times 0.6 \times 0.8 = 0.144$$

$$P = 0.448 + 0.072 + 0.084 + 0.144 = 0.748$$

- (d) (i) Let  $p_n$  be the probability that Michael is successful with his  $n^{\text{th}}$  free throw in the game (and hence  $(1-p_n)$  is the probability that Michael is unsuccessful with his  $n^{\text{th}}$  free throw). Show that  $p_{n+1} = 0.6 + 0.2p_n$ .

$$\begin{aligned} p_{n+1} &= P(S,S) + P(U,S) \\ &= p_n \times 0.8 + (1-p_n)0.6 \\ &= 0.6 + 0.2p_n \end{aligned}$$

- (ii) Assume that  $p$  is Michael's success rate in the long run; that is, for large values of  $n$ , we have  $p_{n+1} \approx p_n \approx p$ . Using the result from part (d) (i) above, or otherwise, show that  $p = 0.75$ .

$$\begin{aligned} p \approx p_n \approx p_{n+1} &= 0.6 + 0.2p_n \\ \Rightarrow 0.8p_n &= 0.6 \\ \Rightarrow p_n &= \frac{0.6}{0.8} = 0.75 = p \end{aligned}$$

- (e) For all positive integers  $n$ , let  $a_n = p - p_n$ , where  $p = 0.75$  as above.

- (i) Use the ratio  $\frac{a_{n+1}}{a_n}$  to show that  $a_n$  is a geometric sequence with common ratio  $\frac{1}{5}$ .

$$\begin{aligned} \frac{a_{n+1}}{a_n} &= \frac{p - p_{n+1}}{p - p_n} \\ &= \frac{0.75 - (0.6 + 0.2p_n)}{0.75 - p_n} \\ &= \frac{0.15 - 0.2p_n}{5(0.15 - 0.2p_n)} = \frac{1}{5} \end{aligned}$$



- (ii) Find the smallest value of  $n$  for which  $p - p_n < 0.00001$ .

$$a_n = p - p_n$$
$$a_1 = p - p_1 = 0.75 - 0.7 = 0.05$$

$$ar^{n-1} = 0.05(0.2)^{n-1} < 0.00001$$

$$(n-1)\ln 0.2 < \ln 0.0001$$

$$\Rightarrow n-1 > \frac{\ln 0.0001}{\ln 0.2} = 5.29$$

$$\Rightarrow n > 6.29$$

$$n = 7$$

- (f) You arrive at a game in which Michael is playing. You know that he has already taken many free throws, but you do not know what pattern of success he has had.

- (i) Based on this knowledge, what is your estimate of the probability that Michael will be successful with his next free throw in the game?

Answer:  $0.75$  or  $p$

- (ii) Why would it **not** be appropriate to consider Michael's subsequent free throws in the game as a sequence of Bernoulli trials?

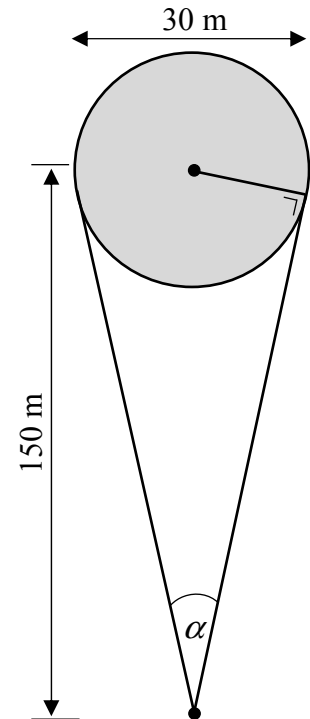
Events not independent

**Question 9**

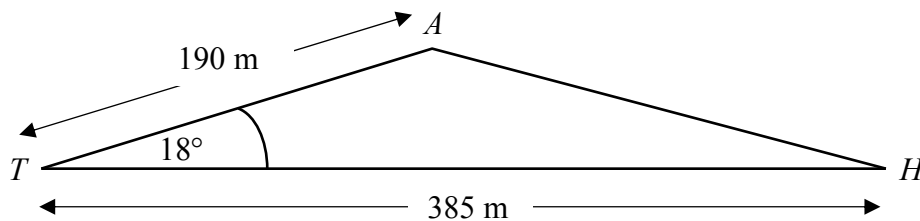
**(45 marks)**

- (a) Joan is playing golf. She is 150 m from the centre of a circular green of diameter 30 m. The diagram shows the range of directions in which Joan can hit the ball so that it could land on the green. Find  $\alpha$ , the measure of the angle of this range of directions. Give your answer, in degrees, correct to one decimal place.

$$\begin{aligned} \sin \frac{1}{2}\alpha &= \frac{15}{150} = 0.1 \\ \Rightarrow \frac{1}{2}\alpha &= 5.739^\circ \\ \Rightarrow \alpha &= 11.478^\circ \\ \alpha &= 11.5^\circ \end{aligned}$$

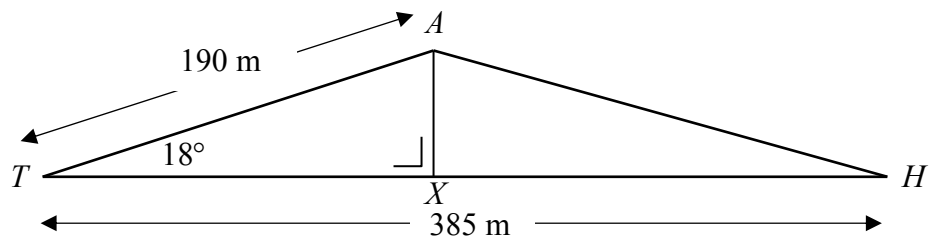


- (b) At the next hole, Joan, at  $T$ , attempts to hit the ball in the direction of the hole  $H$ . Her shot is off target and the ball lands at  $A$ , a distance of 190 metres from  $T$ , where  $|\angle ATH| = 18^\circ$ .  $|TH|$  is 385 metres. Find  $|AH|$ , the distance from the ball to the hole, correct to the nearest metre.



$$\begin{aligned} |AH|^2 &= 190^2 + 385^2 - 2(190)(385)\cos 18^\circ \\ &= 36100 + 148225 - 139139 \cdot 5683 \\ &= 45185.4317 \\ |AH| &= 212.57 = 213 \end{aligned}$$

or



Draw  $AX$  perpendicular to  $TH$

triangle  $ATX$ :  $\sin 18^\circ = \frac{|AX|}{190} \Rightarrow |AX| = 58.71$

$$\cos 18^\circ = \frac{|TX|}{190} \Rightarrow |TX| = 180.7$$

$$\Rightarrow |XH| = 204.3$$

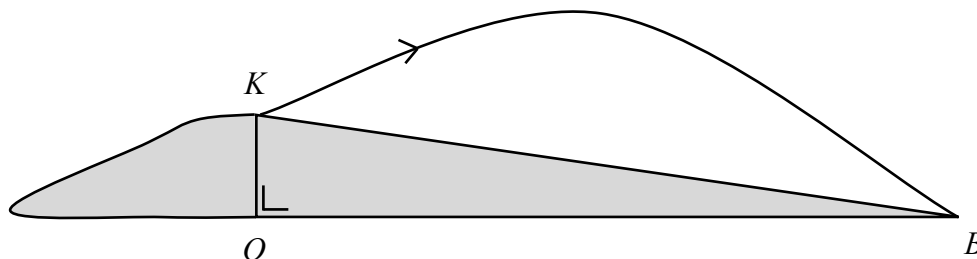
$$\Rightarrow |AH|^2 = (58.71)^2 + (204.3)^2$$

$$\Rightarrow |AH| = 212.566 = 213$$

- (c) At another hole, where the ground is not level, Joan hits the ball from  $K$ , as shown. The ball lands at  $B$ . The height of the ball, in metres, above the horizontal line  $OB$  is given by

$$h = -6t^2 + 22t + 8$$

where  $t$  is the time in seconds after the ball is struck and  $h$  is the height of the ball.



- (i) Find the height of  $K$  above  $OB$ .

$$h = -6t^2 + 22t + 8$$

$$t = 0 \Rightarrow h = 8 \text{ m}$$

- (ii) The horizontal speed of the ball over the straight distance  $[OB]$  is a constant  $38 \text{ m s}^{-1}$ . Find the angle of elevation of  $K$  from  $B$ , correct to the nearest degree.

$$h = 0 \Rightarrow -6t^2 + 22t + 8 = 0$$

$$\Rightarrow (t - 4)(-6t - 2) = 0$$

$$\Rightarrow t = 4, \quad t = -\frac{1}{3}$$

$$t = 4 \Rightarrow |OB| = 38 \times 4 = 152 \text{ m}$$

$$\tan |\angle OBK| = \frac{8}{152} = \frac{1}{19} \Rightarrow |\angle OBK| = 3.01^\circ = 3^\circ$$

- (d) At a later hole, Joan's first shot lands at the point  $G$ , on ground that is sloping downwards, as shown. A vertical tree,  $[CE]$ , 25 metres high, stands between  $G$  and the hole. The distance,  $|GC|$ , from the ball to the bottom of the tree is also 25 metres.

The angle of elevation at  $G$  to the top of the tree,  $E$ , is  $\theta$ , where  $\theta = \tan^{-1} \frac{1}{2}$ .

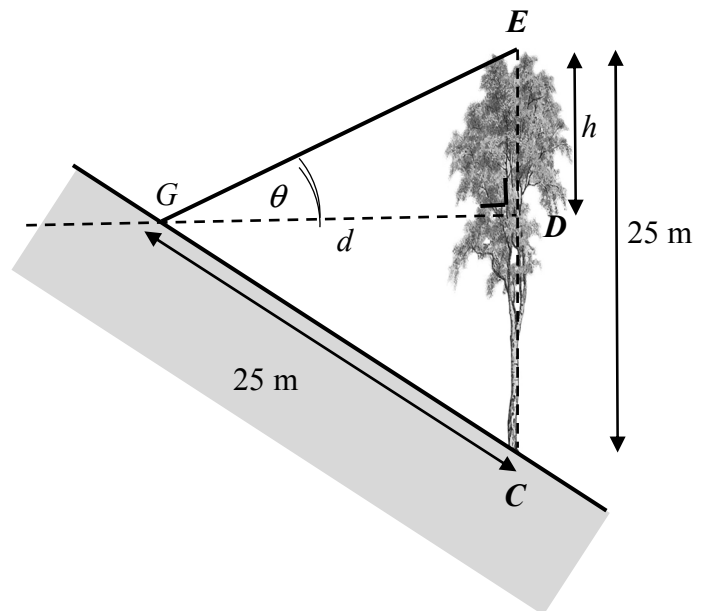
The height of the top of the tree above the horizontal,  $GD$ , is  $h$  metres and  $|GD| = d$  metres.

- (i) Write  $d$  and  $|CD|$  in terms of  $h$ .

$$\tan \theta = \frac{h}{d} = \frac{1}{2}$$

$$\Rightarrow d = 2h$$

$$|CD| = 25 - h$$



- (ii) Hence, or otherwise, find  $h$ .

$$d^2 + |CD|^2 = 25^2$$

$$(2h)^2 + (25 - h)^2 = 25^2$$

$$4h^2 + 625 - 50h + h^2 = 625$$

$$5h^2 - 50h = 0$$

$$h = 0, \quad h = 10$$

$$h = 10 \text{ m}$$

or

$$\theta = \tan^{-1} \frac{1}{2} = 26.565^\circ$$

$$\Rightarrow |GED| = 63.435^\circ$$

$$\Rightarrow |CGE| = 63.435^\circ$$

$$\Rightarrow |CGD| = 63.435^\circ - 26.565^\circ = 36.87^\circ$$

$$\sin 36.87 = \frac{25 - h}{25} = 0.6$$

$$\Rightarrow 25 - h = 15$$

$$\Rightarrow h = 10 \text{ m}$$

or

$$|\angle GCE = 53.14^\circ| \Rightarrow \sin 53.14^\circ = \frac{2h}{25}$$

$$\Rightarrow 0.8 = \frac{2h}{25} \Rightarrow h = 10 \text{ m}$$

## Marking Scheme – Paper 2, Section A and Section B

### Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales		0, 2, 5	0, 2, 4, 5	0,2,3,4,5	
10 mark scales		0, 5, 10	0, 4, 8, 10	0, 2, 5, 8, 10	
15 mark scales			0, 5, 12, 15	0, 4, 7, 11, 15	
20 mark scales					
25 mark scales					

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

### Marking scales – level descriptors

#### A-scales (two categories)

- incorrect response
- correct response

#### B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

#### C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

#### D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

#### E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

**NOTE:** In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded.

Rounding and units penalty to be applied only once in each section (a), (b), (c) etc.

Throughout the scheme indicate by use of \* where an arithmetic error occurs.

## Summary of mark allocations and scales to be applied

### *Section A*

#### **Question 1**

(a)	10C
(b)(i)+(ii)	10C
(c)	5C

#### **Question 2**

(a)	10C
(b)	10D
(c)	5C

#### **Question 3**

(a)	10D
(b)	10C
(c)	5D

#### **Question 4**

(a)	5B
(b)(i)	5C
(b)(ii)	10C
(c)	5C

#### **Question 5**

(a)	15D
(b)	10D

#### **Question 6**

(a)	5C
(b) Diag	5B
Const	5B
Proof	10C

### *Section B*

#### **Question 7**

(a)	15C
(b)	15C
(c)(i)	5C
(c)(ii)	5D

#### **Question 8**

(a)	10C
(b)	10C
(c)	15D
(d)(i)	5C
(d)(ii)	10B
(e)(i)	5C
(e)(ii)	5C
(f)	5B

#### **Question 9**

(a)	10C
(b)	10C
(c)(i)	5B
(c)(ii)	10C
(d)(i)	5B
(d)(ii)	5D

## Section A

### Question 1

(25 marks)

**(a) Scale 10C (0, 4, 8, 10)**

*Low Partial Credit:*

- At least one other correct entry
- Partially correct table with at least 5 correct totals or couples

*High Partial Credit:*

- Five or more correct entries including at least one other loss and one other win
- Table correctly completed with totals or couples but no indication of W or L

**(b)(i)(ii) Scale 10C (0, 4, 8, 10)**

*Low Partial Credit:*

- Favourable outcomes identified
- (i) correct only ( $\frac{10}{36}, \frac{5}{18}, 0.2\dot{7}, 0.28, 0.3$ )

*High Partial Credit:*

- (i) omitted or of no merit but (ii)  $\left(\frac{13}{18}\right)^3$

**(c) Scale 5C (0, 2, 4, 5)**

*Low Partial Credit:*

- Relevant binomial formula with some substitution
- Identifies  $p^7$  or  $(1-p)^3$  or  $(1-p)^2$  or  $1-p$
- Listing at least any two of the ten throws

*High Partial Credit*

- Probability of two wins in nine throws



## Question 2

(25 marks)

### (a) Scale 10C (0, 4, 8, 10)

*Low Partial Credit:*

- Relevant formula with or without substitution
- $\frac{1}{\sqrt{n}}$  with further work

*High Partial Credit*

- $1.96 \frac{\sigma}{\sqrt{n}}$  evaluated

### (b) Scale 10D (0, 2, 5, 8, 10)

*Low Partial Credit:*

- One relevant step e.g. null hypothesis or alternative hypothesis stated
- Some work towards finding  $z$
- Mention of  $\pm 1.96$

*Mid Partial Credit:*

- $z$  calculated
- Either null or alternative hypothesis stated and relevant work towards finding  $z$
- Confidence interval from (a) and either null or alternative hypothesis stated
- Confidence interval based on 100 (i.e. 89.94, 98.06) and either null or alternative hypothesis stated.

*High Partial Credit:*

- $z$  calculated and compared to  $\pm 1.96$  but:
  - Not stating null hypothesis and / or alternative hypothesis correctly
  - Not accepting or rejecting hypothesis
  - Incorrect conclusion for hypothesis
- Incorrect use of 94 and confidence interval
- Incorrect use of 90.45 and confidence interval

### (c) Scale 5C (0, 2, 4, 5)

*Low Partial Credit:*

- Effort at finding  $P(z < -1.71)$

*High Partial Credit:*

- $p$  value correct
- Not contextualising answer correctly

**Question 3**

**(25 marks)**

**(a) Scale 10D (0, 2, 5, 8,10)**

*Low Partial Credit:*

- Slope  $AB$  or  $l_1$

*Mid Partial Credit:*

- Both slopes found

*High Partial Credit:*

- Slopes linked to perpendicularity

**(b) Scale 10C (0, 4, 8, 10)**

*Low Partial Credit:*

- Relevant formula with some correct substitution

*High Partial Credit*

- Substitution into formula fully correct

**(c) Scale 5D (0, 2, 3, 4, 5)**

*Low Partial Credit:*

- Relevant formula with some correct substitution

*Mid Partial Credit:*

- One value for  $k$  found
- Work indicating two values for  $k$

*High Partial Credit:*

- Both values of  $k$
- Positive value for  $k$  evaluated and distance calculated

**Question 4**

**(25 marks)**

**(a) Scale 5B (0, 2, 5)**

*Partial Credit:*

- Centre or radius

**(b)(i) Scale 5C (0, 2, 4, 5)**

*Low Partial Credit:*

- Formula for ratio with some correct substitution
- Effort at setting up translation

*High Partial Credit:*

- Substitution into ratio formula fully correct
- One ordinate only found
- Correct answer without supporting work

**(b)(ii) Scale 10C (0, 4, 8, 10)**

*Low Partial Credit:*

- Identifies centre
- Identifies radius

*High Partial Credit:*

- Equation of circle formed but error in substitution

**(c) Scale 5C (0, 2, 4, 5)**

*Low Partial Credit:*

- Slope  $AB$  or slope of tangent
- Some correct substitution into relevant formula

*High Partial Credit:*

- Equation of line fully substituted

**Question 5****(25 marks)****(a) Scale 15D (0, 4, 7, 11, 15)***Low Partial Credit:*

- Tan function in terms of Sine and Cosine

*Mid Partial Credit:*

- $\sin(A+B)$  or  $\cos(A+B)$  expanded
- Numerator or denominator in fraction form (method 2)

*High Partial Credit:*

- Numerator and denominator divided by  $\cos A \cdot \cos B$  (Method 1)
- Both numerator and denominator expressed in form of a single fraction (Method 2)

**(b) Scale 10D (0, 2, 5, 8, 10)***Low Partial Credit:*

- One value for  $3x$

*Mid Partial Credit:*

- One value for  $x$
- Two or more values for  $3x$

*High Partial Credit:*

- Three or more values for  $x$

**Question 6****(25 marks)****(a) Scale 5C (0, 2, 4, 5)***Low Partial Credit:*

- Some relevant calculation
- One side bisected
- One midpoint indicated

*High Partial Credit:*

- One median drawn

**(b) Diagram / Given : Scale 5B (0, 2, 5)***Partial Credit:*

- Effort at *Diagram* or *Given*

**Construction: Scale 5B (0, 2, 5)***Partial Credit:*

- Construction attempted (diagram and/or description)

**Proof: Scale 10C (0, 4, 8, 10)***Low Partial Credit:*

- More than one critical step omitted but still some substantial work of merit

*High Partial Credit:*

- Proof completed with one critical step omitted

## Section B

### Question 7

(40 marks)

**(a) Scale 15C (0, 5, 12, 15)**

*Low Partial Credit:*

- $BT$  drawn correctly
- Pythagoras formula with some correct substitution
- Recognising  $|\angle ATB| = 90^\circ$

*High Partial Credit:*

- Pythagoras formula fully substituted

**(b) Scale 15C (0, 5, 12, 15)**

*Low Partial Credit:*

- Indicates two areas
- Effort at area of rectangle only
- Effort at area of triangle only

*High Partial Credit:*

- Area of triangle correct
- Area of rectangle correct

**(c)(i) Scale 5C (0, 2, 4, 5)**

*Low Partial Credit:*

- $\tan \angle HAB = \frac{160}{60}$  or equivalent in sin or cos

*High Partial Credit:*

- $|\angle HAB|$  in degrees.

**(c)(ii) Scale 5D (0, 2, 3, 4, 5)**

*Low Partial Credit:*

- Effort at area of one region

*Mid Partial Credit:*

- Area of one sector with correct substitution

*High Partial Credit:*

- Area of two sectors with substitution correct in both.

**Question 8****(65 marks)****(a) Scale 10C (0, 4, 8, 10)***Low Partial Credit:*

- One correct probability

*High Partial Credit:*

- Identifies all three probabilities correctly
- Three probabilities multiplied of which two are correct

**(b) Scale 10C (0, 4, 8, 10)***Low Partial Credit:*

- One correct probability

*High Partial Credit:*

- Identifies all three probabilities correctly
- Three probabilities multiplied of which two are correct

**(c) Scale 15D (0, 4, 7, 11, 15)***Low Partial Credit:*

- Lists one new way

*Mid Partial Credit:*

- Full listing only
- One new probability

*High Partial Credit:*

- Sum of three probabilities
- Identifies all four probabilities correctly

**(d) (i) Scale 5C (0, 2, 4, 5)***Low Partial Credit:*

- Indicates  $P(S, S)$  and/or  $P(U, S)$  or equivalent

*High Partial Credit:*

- Substitution into equation for  $p_{n+1}$

**(d) (ii) Scale 10B (0, 5, 10)***Partial Credit:*

- Partial substitution into equation

**(e)(i) Scale 5C (0, 2, 4, 5)***Low Partial Credit:*

- $a_{n+1}$  in terms of  $p$  and  $p_{n+1}$
- $\frac{a_{n+1}}{a_n}$  in terms of  $p$ ,  $p_n$ , and  $p_{n+1}$

*High Partial Credit:*

- $\frac{a_{n+1}}{a_n}$  substituted

**(e)(ii) Scale 5C (0, 2, 4, 5)**

*Low Partial Credit:*

- $a_1$  in numerical form

*High Partial Credit:*

- $ar^{n-1}$  substituted
- $a_7$  evaluated without checking  $a_6$

**(f) Scale 5B (0, 2, 5)**

*Partial Credit:*

- (i) correct only or (ii) correct only

**Question 9**

**(45 marks)**

**(a) Scale 10C (0, 4, 8, 10)**

*Low Partial Credit:*

- Effort at expressing sine function in terms of 15 and 150
- Finds third side of triangle and makes effort to find an angle

*High Partial Credit:*

- Half angle found

**(b) Scale 10C (0, 4, 8, 10)**

*Low Partial Credit:*

- Cosine Rule with some correct substitution
- Effort at calculating  $|AX|$  or  $|TX|$

*High Partial Credit:*

- Cosine Rule substituted correctly
- Finds  $|AX|$  and formulates for  $|TX|$  (or vice versa)

**(c)(i) Scale 5B (0, 2, 5)**

*Partial Credit:*

- $t = 0$  indicated

Accept  $h = 8$  m without work

**(c)(ii) Scale 10C (0, 4, 8, 10)**

*Low Partial Credit:*

- $h = 0$  indicated

*High Partial Credit:*

- $|OB|$  found for positive value for  $t$

**(d)(i) Scale 5B (0, 2, 5)**

*Partial Credit:*

- $\frac{h}{d} = \frac{1}{2}$
- $|CD| = 25 - h$

**(d)(ii) Scale 5D (0, 2, 3, 4, 5)**

*Low Partial Credit:*

- Pythagoras with some correct substitution.
- Effort at evaluating  $\theta$

*Mid Partial Credit:*

- Pythagoras correctly substituted
- $\tan^{-1} \frac{1}{2}$  evaluated

*High Partial Credit:*

- Quadratic equation expanded correctly
- $\sin \angle CGD = \frac{|CD|}{|GC|}$  with  $|\angle CGD|$  calculated
- $\sin \angle GCE = \frac{|GD|}{|GC|}$  with  $|\angle GCE|$  calculated



## Marcanna breise as ucht freagairt trí Ghaeilge

### (Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g.  $198 \text{ marc} \times 5\% = 9.9 \Rightarrow$  bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle  $[300 - \text{bunmharc}] \times 15\%$ , agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0

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